Math 371 - Fall 2011 - 10/6/2011 - Quiz #8 **Score:** ______ out of 10.

Name: key Seat:

Show all work clearly and in order. Please box your answers. 10 minutes.

For each of the following linear differential equations: (a) find the general solution to the associated homogeneous differential equation, and (b) determine the FORM of a particular solution. DO NOT SOLVE FOR THE UNDETERMINED COEFFICIENTS.

1. $y'' - 3y' + 2y = 3x.$

Aux. eqn.: $m^2 - 3m + 2 = 0$
$(m-2)(m-1) = 0 \rightarrow m = 2$ or $m = 1$

(a) $y_c = \boxed{c_1 e^{2x} + c_1 e^{x}}$ please see below

(b) $y_p = \boxed{Ax + B}$ please see below

2. $y'' - 2y' + y = e^x.$

Aux eqn.: $m^2 - 2m + 1 = 0$
$(m-1)(m-1) \rightarrow m = 1$ or $m = 1$

(a) $y_c = \boxed{c_1 e^{x} + C_2 x e^{x}}$ please see below

(b) $y_p = \boxed{Ax^2 e^{x}}$ please see below

3. $y'' - 2y' + y = e^x + x^2.$

Aux eqn. $m^2 - 2m + 1 = 0 \rightarrow m = 1$ or $m = 1$
$(m-1)(m-1) = 0$

(a) $y_c = \boxed{c_1 e^{x} + C_2 x e^{x}}$ please see below

(b) $y_p = \boxed{Ax^2 e^{x} + Bx^2 + Cx + D}$ please ~~see~~ below

4. $y'' - 2y' + 5y = \cos(3x).$

$m^2 - 2m + 5 = 0 \rightarrow m = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(5)}}{2} = \frac{2 \pm \sqrt{-16}}{2} = 1 \pm \frac{4i}{2} = 1 \pm 2i$

$\alpha = 1,\ \beta = 2$

(a) $y_c = \boxed{e^{x}[c_1 \cos(2x) + C_2 \sin(2x)]}$ please see below

(b) $y_p = \boxed{A\cos(3x) + B\sin(3x)}$ please see below

~~since g(x) = cos(3x) and $y_1 = e^x\cos(2x)$ $y_2 = e^x\sin(2x)$~~

~~neither is a constant multiple~~

~~g(x) is NOT a constant multiple~~

5. $y'' - 2y' + 5y = \cos(2x).$

$m^2 - 2m + 5 = 0$ (same as in 4.) $m = 1 \pm 2i$
$\alpha = 1,\ \beta = 2$

(a) $y_c = \boxed{e^{x}[c_1 \cos(2x) + C_2 \sin(2x)]}$ please see below

(b) $y_p = \boxed{A\cos(2x) + B\sin(2x)}$ please see below

~~since g(x) = cos(2x) and $y_1 = e^x\cos(2x)$ $y_2 = e^x\sin(2x)$~~

$\boxed{\text{LOOK AT TABLE 3.4.1 on p123}}$

(1) (b) since $y_c = c e^{2x} + c_1 e^{x}$

we have $y_1 = e^{2x}$ AND $y_2 = e^{x}$

Now, since $g(x) = 3x$ the FORM of y_p from Table 3.4.1 #2 is:

$$\boxed{y_p = Ax + B}$$

and since <u>no part of y_p is equal to y_1 or y_2 we do not need to adjust y_p</u>

(i.e., no function in this form of y_p is a solution of the associated homogeneous differential equation.)

(2) (b) here $y_1 = e^{x}$

$y_2 = xe^{x}$

$g(x) = e^{x}$

From Table 3.4.1 #7 the form of y_p is

$$y_p = Ae^{x}$$

but this contains y_1 in it so we multiply by x:

$$y_p = Axe^{x}$$

but this also contains y_2 (another solution to the associated homogeneous DE) so

$$\boxed{y_p = Ax^2e^{x}}$$

③ (b) Here $y_1 = e^x$

$y_2 = xe^x$

$$g(x) = \underbrace{e^x}_{g_1(x)} + \underbrace{x^2}_{g_2(x)}$$

we need to find y_{p_1} for $g_1(x)$ AND y_{p_2} for $g_2(x)$ } then $y_p = y_{p_1} + y_{p_2}$

looking at $g_1(x)$ and Table 3.4.1 # 7

$$y_{p_1} = Ae^x$$

~~But~~ However, we need to adjust this because this contains y_1 so

$$y_{p_1} = Axe^x,$$

but this contains y_2. Hence,

$$y_{p_1} = Ax^2e^x$$

looking at $g_2(x)$ and Table 3.4.1 # 3

$$y_{p_2} = Bx^2 + Cx + D$$

and no part of this contains y_1 and y_2

Thus, $\boxed{y_p = y_{p_1} + y_{p_2} = Ax^2e^x + Bx^2 + Cx + D}$

(4) (a) $y_c = e^x\left[c_1 \cos(2x) + c_2 \sin(2x)\right]$

OR → $= c_1 \underbrace{e^x\cos(2x)}_{y_1} + c_2 \underbrace{e^x\sin(2x)}_{y_2}$

(b)

so $y_1 = e^x\cos(2x)$

$y_2 = e^x\sin(2x)$

$g(x) = \cos(3x)$

looking at table 3.4.1 #6

$$\boxed{y_p = A\cos(3x) + B\sin(3x)}$$

and no part of this contains y_1 or y_2

so we do not need to adjust!

(5) (a) $y_c = e^x\left[c_1 \cos(2x) + c_2 \sin(2x)\right]$

OR → $= c_1 \underbrace{e^x\cos(2x)}_{y_1} + c_2 \underbrace{e^x\sin(2x)}_{y_2}$

(b) so $y_1 = e^x\cos(2x)$

$y_2 = e^x\sin(2x)$

$g(x) = \cos(2x)$

looking at table 3.4.1 #6

$$\boxed{y_p = A\cos(2x) + B\sin(2x)}$$

and no part of y_p contains y_1 or y_2, so no adjustment!