Quiz # 7 - HW Quiz

3.3 p. 118 (# 32)

Solve the given initial-value problem:

$$4y'' - 4y' - 3y = 0, \quad y(0) = 1, \quad y'(0) = 5$$

SOL: 2nd order, linear DE w/ constant coef., homogeneous.

Aux equation: $4m^2 - 4m - 3 = 0$

$$m = \frac{4 \pm \sqrt{(-4)^2 - 4(4)(-3)}}{2(4)} = \frac{4 \pm \sqrt{16 + 48}}{8}$$

$$= \frac{4 \pm \sqrt{64}}{8}$$

$$= \frac{4 \pm 8}{8}$$

$$= \frac{1}{2} \pm 1$$

$m = -\frac{1}{2}$ or $m = \frac{3}{2}$

distinct real roots (case I on p113)

So

$$y = c_1 e^{-x/2} + c_2 e^{3x/2}$$

Now substitute the initial conditions:

$$y(0) = 1 = c_1 e^0 + c_2 e^0 = c_1 + c_2$$

so $c_1 = 1 - c_2$

$$y' = c_1\left(-\frac{1}{2}\right)e^{-x/2} + c_2\left(\frac{3}{2}\right)e^{3x/2}$$

$$y'(0) = -\frac{c_1}{2} e^0 + \frac{3c_2}{2} e^0 = -\frac{c_1}{2} + \frac{3c_2}{2} = 5$$

$-\frac{(1-c_2)}{2} + \frac{3c_2}{2} = 5 \Rightarrow \frac{-1 + c_2 + 3c_2}{2} = 5 \Rightarrow -1 + 4c_2 = 10 \Rightarrow c_2 = \frac{11}{4}, \; c_1 = 1 - \frac{11}{4} = \frac{-7}{4} \longrightarrow$

So $\boxed{y = \frac{-7}{4} e^{-x/2} + \frac{11}{4} e^{3x/2}}$