Math 371 - Fall 2011 - 9/27/2011 - Quiz #6 Score: ______ out of 10.

Name: Key Seat:

Show all work clearly and in order. Please box your answers. 10 minutes.

PICK ONE OF THE FOLLOWING:

Please indicate which one you do NOT want me to grade by putting an X through it, otherwise I will grade the first one worked on:

1. Verify $y_1(x) = e^x$ and $y_2(x) = e^{2x}$ form a fundamental set of solutions of the differential equation:

$$y'' - 3y' + 2y = 0 \quad \text{on } (-\infty, \infty).$$

(i) $\left.\begin{array}{l} y_1(x) = e^x \\ y_1'(x) = e^x \\ y_1''(x) = e^x \end{array}\right\}$ $y_1'' - 3y_1' + 2y_1 = e^x - 3e^x + 2e^x = 0$ ✓

so $y_1(x)$ is a solution.

(ii) $\left.\begin{array}{l} y_2(x) = e^{2x} \\ y_2(x) = 2e^{2x} \\ y_3(x) = 4e^{2x} \end{array}\right\}$ $y_2'' - 3y_2' + 2y_2 = 4e^{2x} - 3(2e^{2x}) + 2e^{2x} = 0$ ✓

so $y_2(x)$ is a solution.

(iii) $W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^x & e^{2x} \\ e^x & 2e^{2x} \end{vmatrix} = e^x(2e^{2x}) - e^{2x}e^x$

$= 2e^{3x} - e^{3x} = e^{3x} \neq 0$ on $(-\infty, \infty)$

So y_1 and y_2 are linearly independent. Hence, by (i), (ii) and (iii) y_1 and y_2 form a fund. set of solutions.

2. The function $y_1(x) = e^{2x}$ is a solution of

$$y'' - 4y' + 4y = 0. \tag{1}$$

Use reduction of order to find a second solution $y_2(x)$ of (1).

$y_2 = u y_1 = u e^{2x}$

$y_2' = u(2e^{2x}) + e^{2x}u'$

$y_2'' = u(4e^{2x}) + (2e^{2x})u' + e^{2x}u'' + u'(2e^{2x})$

$= u''e^{2x} + 4e^{2x}u' + 4ue^{2x}$

$y_2'' - 4y_2' + 4y_2 = 0$

$(u''e^{2x} + 4u'e^{2x} + 4ue^{2x}) - 4(2ue^{2x} + u'e^{2x}) + 4ue^{2x} = 0$

$e^{2x}(u'' \cancel{+4u'} \cancel{+4u} \cancel{-8u} \cancel{-4u'} \cancel{+4u}) = 0$, since $e^{2x} \neq 0$, it must be that $u'' = 0 \Rightarrow u' = C \Rightarrow u = Cx + D$

if $C = 1$ and $D = 0$ a possible second solution is $y_2 = xe^{2x}$ (there are many answers)

$y_2(x) = \boxed{xe^{2x}}$

N.B., the gen. sol. would be $y = C_1y_1 + C_2y_2 = C_1e^{2x} + C_2((Cx+D)e^{2x}) = E_1e^{2x} + \underline{E_2xe^{2x}}$ so this is y_2