Quiz #5 – HW Quiz.

2.5 p 64 # 16

solve $\frac{dy}{dx} - y = e^x y^2$

by using an appropriate substitution.

SOL : Notice that this is a Bernoulli Equation (p.63) equation (4). $\left(\frac{dy}{dx} + P(x)y = f(x)\, y^n \right)$

where $P(x) = -1$, $f(x) = e^x$ and $y^n = y^2$

meaning $n = 2$.

substitute $u = y^{1-n} = y^{1-2} = y^{-1} = \frac{1}{y}$

that is $y = u^{-1}$

so $\frac{dy}{dx} = -u^{-2}\frac{du}{dx}$

Hence, $\frac{dy}{dx} - y = e^x y^2$ becomes :

$$\left(-u^{-2}\frac{du}{dx}\right) - \left(u^{-1}\right) = e^x\left(u^{-1}\right)^2$$

$$\frac{1}{-u^2}\frac{du}{dx} - \frac{1}{u} = \frac{e^x}{u^2}$$

multiply through by $-u^2$:

$$\frac{-u^2}{-u^2}\frac{du}{dx} - \frac{-u^2}{u} = \frac{-u^2 e^x}{u^2}$$

$$\frac{du}{dx} + u = -e^x$$

1st order linear.

Integrating factor (I.F.) $e^{\int 1 dx} = e^x$

multiply by I.F.

$$\underbrace{e^x\left[\frac{du}{dx} + u\right]} = e^x(-e^x) = -e^{2x}$$

$$\frac{d}{dx}\left[e^x u\right] = -e^{2x}$$

$$e^x u = -\int e^{2x}dx = -\frac{e^{2x}}{2} + C$$

$$u = -\frac{e^x}{2} + \frac{C}{e^x}$$

substituting for y (recall $y = \frac{1}{u} \Rightarrow u = \frac{1}{y}$)

$$\boxed{\frac{1}{y} = \frac{-e^x}{2} + \frac{C}{e^x}}$$