Math 371 - Fall 2011 - 9/20/2011 - Quiz #4 **Score:** ______ out of 10.

Name: Key Seat:

Show all work clearly and in order. Please box your answers. 10 minutes.

1. Consider the following differential equation:

$$\underbrace{y(x+y+1)}_{M(x,y)}dx + \underbrace{(x+2y)}_{N(x,y)}dy = 0. \tag{1}$$

(a) Show that (1) is NOT an exact equation. DO NOT SOLVE THE DIFFERENTIAL EQUATION!

Let $M(x,y) = y(x+y+1) = yx + y^2 + y$ AND Let $N(x,y) = x+2y$

Notice that

$$M_y = \frac{\partial M}{\partial y} = x + 2y + 1$$

$\updownarrow$ Not equal

$$N_x = \frac{\partial N}{\partial x} = 1$$

since $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$, (1) is NOT exact.

(b) Give an integrating factor that will turn (1) into an exact equation. SIMPLIFY! DO NOT SOLVE THE DIFFERENTIAL EQUATION!

Notice that $\frac{M_y - N_x}{N} = \frac{(x+2y+1)-(1)}{x+2y} = \frac{x+2y}{x+2y} = 1$ ⟸ function of just x

so Integrating Factor: $\mu(x) = e^{\int \frac{M_y - N_x}{N}dx} = e^{\int 1 dx} = e^x$

$\mu = \boxed{e^x}$

2. Consider the following differential equation:

$$\underbrace{(y^2 + yx)}_{M(x,y)}dx + \underbrace{x^2}_{N(x,y)}dy = 0. \tag{2}$$

(a) Show that (2) is homogeneous. DO NOT SOLVE THE DIFFERENTIAL EQUATION!

Let $M(x,y) = y^2 + yx$ AND let $N(x,y) = x^2$

Notice that

$$M(tx,ty) = (ty)^2 + (ty)(tx) = t^2y^2 + t^2xy = t^2(y^2+xy) = t^2 M(x,y)$$

$$N(tx,ty) = (tx)^2 = t^2x^2 = t^2 N(x,y)$$

so both M and N are homogeneous functions of degree 2, Hence, (2) is homogeneous.

(b) Give a substitution that will allow you to solve (2). (No work is needed here.) DO NOT SOLVE THE DIFFERENTIAL EQUATION!

$u = \boxed{y/x}$ (or $u = x/y$)