Quiz #3 – HW Quiz

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Solve the given initial value problem (IVP):

$$\begin{cases} (e^x + y)\,dx + (2 + x + ye^y)\,dy = 0 \\ y(0) = 1 \end{cases}$$

SOLUTION:

here $M(x,y) = e^x + y$

$N(x,y) = 2 + x + ye^y$

(1) Check if exact: $\frac{\partial M}{\partial y} = 1 = \frac{\partial N}{\partial x}$ ✓

(2) $f(x,y) = \int M(x,y)\,dx + g(y)$

$= \int (e^x + y)\,dx + g(y)$

$= e^x + yx + g(y)$

(2) $\frac{\partial}{\partial y} f(x,y) = 0 + x + g'(y) = \underbrace{2 + x + ye^y}_{N(x,y)}$

so $g'(y) = 2 + ye^y$

(3) $\int g'(y)\,dy = \int (2 + ye^y)\,dy = 2y + \int ye^y\,dy$

$u = y$ $dv = e^y\,dy$

$du = dy$ $v = e^y$

$= 2y + (ye^y - \int e^y\,dy)$

$= 2y + ye^y - e^y + C$

$= g(y)$

(4) plug $g(y)$ into $f(x,y)$ from (2)

↓

$$f(x,y) = e^x + yx + 2y + ye^y - e^y + C$$

(5) solution of D.E. $\quad f(x,y) = D$

$$e^x + yx + 2y + ye^y - e^y + C = D$$

$$e^x + xy + 2y + ye^y - e^y = E$$

(6) solve IVP. by substituting initial values.

$$y(0) = 1$$

so substitute $\quad x = 0 \quad$ and $\quad y = 1$
to find E.

$$e^0 + 0(1) + 2(1) + (1)e^1 - e^1 = E$$

$$1 + 0 + 2 + e - e = E$$

$$E = 3$$

(7) solution to IVP:

$$\boxed{e^x + xy + 2y + ye^y - e^y = 3}$$