Math 371 - Fall 2011 - 9/13/2011 - Quiz #2 Score: _____ out of 10.

Name: Key Seat:

Show all work clearly and in order. Please box your answers. 10 minutes.

PICK ONE OF THE FOLLOWING:

Please indicate which one you do NOT want me to grade by putting an X through it, otherwise I will grade the first one worked on:

1. Find an implicit AND explicit solution of the following initial-value problem:

$$\frac{dx}{dt} = 2(x^2+1), \qquad x\left(\frac{\pi}{2}\right) = 0$$

This D.E. is seperable, so: $\frac{dx}{x^2+1} = 2\,dt \Rightarrow \int \frac{dx}{x^2+1} = \int 2\,dt$

$\tan^{-1}(x) + C_1 = 2t + C_2$

$\tan^{-1}(x) = 2t + C$

Substitute $t = \frac{\pi}{2}$ and $x = 0$: $\tan^{-1}(0) = 2\left(\frac{\pi}{2}\right) + C$

$0 = \pi + C$

$C = -\pi$

Implicit Solution: $\tan^{-1}(x) = 2t - \pi$

Explicit Solution: $x = \tan(2t - \pi)$

2. (a) Find the general explicit solution of

$$(x+1)\frac{dy}{dx} + (x+2)y = 2xe^{-x}.$$

(b) ♠ Give the largest interval over which the general solution is defined.

(c) ♠ Determine whether there are any transient terms in the general solution.

This is a 1st order linear O.D.E. Put into standard form: $\frac{dy}{dx} + \underbrace{\frac{x+2}{x+1}}_{P(x)} y = \frac{2xe^{-x}}{x+1}$

Integrating Factor (I.F.): $e^{\int P(x)dx} = e^{\int \left(\frac{x+2}{x+1}\right)dx} = e^{\int \left(\frac{x+1+1}{x+1}\right)dx} = e^{\int \left(\frac{x+1}{x+1} + \frac{1}{x+1}\right)dx} = e^{\int \left(1 + \frac{1}{x+1}\right)dx}$

$= e^{x+\ln|x+1|} = e^x e^{\ln|x+1|} = e^x|x+1| = e^x(x+1)$ if $x+1 > 0$ (i.e., $x > -1$)

Multiply ODE by I.F.: $e^x(x+1)\left[\frac{dy}{dx} + \frac{x+2}{x+1}y\right] = e^x(x+1)\cdot\frac{2xe^{-x}}{(x+1)} \Rightarrow \frac{d}{dx}\left[e^x(x+1)y\right] = 2x.$

Integrate Both Sides: $e^x(x+1)y = \frac{2x^2}{2} + C$

Solve for y:

Explicit Solution: $y = \frac{x^2}{e^x(x+1)} + \frac{C}{e^x(x+1)}$

(b): $x > -1$

(c): The entire solution is transient. (All of the terms!)