Quiz #16 - Homework Quiz.

4.5 [scribbled out] p 233 #9

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use the Laplace transform to solve the following IVP:

$$\begin{cases} y'' + 4y' + 5y = \delta(t-2\pi) \\ y(0) = 0 \\ y'(0) = 0 \end{cases}$$

Sol:

$$\mathcal{L}\{y'' + 4y' + 5y\} = \mathcal{L}\{\delta(t-2\pi)\}$$

$$\mathcal{L}\{y''\} + 4\mathcal{L}\{y'\} + 5\mathcal{L}\{y\} = \mathcal{L}\{\delta(t-2\pi)\}$$

$$(s^2Y(s) - sy(0) - y'(0)) + 4(sY(s) - y(0)) + 5Y(s) = e^{-s(2\pi)}$$

$$s^2Y(s) - 0 - 0 + 4sY(s) - 0 + 5Y(s) = e^{-2\pi s}$$

$$Y(s)(s^2 + 4s + 5) = e^{-2\pi s}$$

$$Y(s) = \frac{e^{-2\pi s}}{s^2 + 4s + 5}$$

complete the square: $s^2 + 4s + 5 = (s+r)^2 + c$

$= s^2 + 2sr + r^2 + c$

$2r = 4 \Rightarrow r = 2 \Rightarrow r^2 = 4$

so $s^2 + 4s + 5$

$= s^2 + 4s + 4 - 4 + 5$

$= (s+2)^2 + 1$

$$Y(s) = \frac{1}{(s+2)^2 + 1} e^{-2\pi s}$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{ \underbrace{\frac{1}{(s+2)^2+1}}_{F(s)} \underbrace{e^{-2\pi s}}_{e^{-as}} \right\}$$ so $a = 2\pi$

so $F(s) = \dfrac{1}{(s+2)^2+1}$

if $K=1$ and $a=-2$ then by #14

$$f(t) = e^{-2t}\sin(t)$$

$$f(t-2\pi) = e^{-2(t-2\pi)}\sin(t-2\pi)$$

$$= e^{-2(t-2\pi)}\sin(t)$$

by formula #10:

$$y(t) = f(t-2\pi)\,\mathcal{U}(t-2\pi)$$

$$\boxed{y(t) = e^{-2(t-2\pi)}\sin(t)\,\mathcal{U}(t-2\pi)}$$