Math 371 - Fall 2011 - 11/22/2011 - Quiz #15 **Score:** _____ out of 10.

Name: key Seat:

Show all work clearly and in order. Please box your answers. Due 11/29/2011, 8:00AM

1. Evaluate the following (using any correct method).

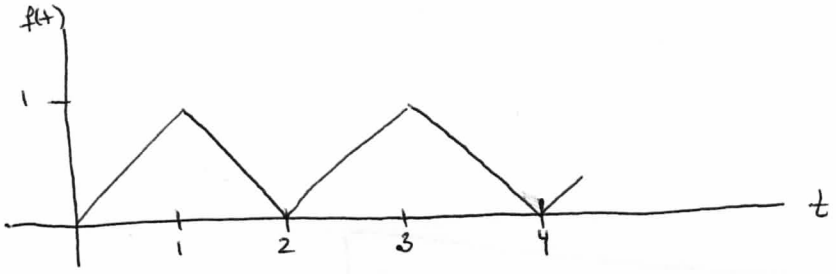
(a) $\mathcal{L}\{t^2 * te^t\} = \mathcal{L}\{t^2\}\mathcal{L}\{te^t\} = \left(\frac{2!}{s^3}\right)\left(\frac{1!}{(s-1)^2}\right) = \boxed{\frac{2}{s^3(s-1)^2}}$

by #3 and #13

(b) $\mathcal{L}\left\{\int_0^t \underbrace{e^{-\tau}\cos(\tau)}_{f(\tau)}\, d\tau\right\}$ so $f(t) = e^{-t}\cos(t) \overset{\#15}{\Longrightarrow} F(s) = \frac{s+1}{(s+1)^2+1}$

$\overset{\#19}{=} \frac{F(s)}{s} = \boxed{\frac{s+1}{s\left((s+1)^2+1\right)}}$

2. Write your solution to problem 52 on p.229. (You did this problem in homework #15).



Find the Laplace transform of f(t)

SOL: The period is 2, so $T=2$.

using Thm. 4.4.3 on p226: $\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-sT}}\int_0^T e^{-st} f(t)\, dt$

$= \frac{1}{1-e^{-2s}}\int_0^2 e^{-st} f(t)\, dt$

what is $f(t)$ on the interval $[0,2]$?

Notice that $f(t) = \begin{cases} t & \text{if } 0 \le t < 1 \\ 2-t & \text{if } 1 \le t < 2 \end{cases}$ AND $f(t+2) = f(t)$ for all t.

Hence, we can simplify the integral as follows:

$$\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-2s}}\left[\int_0^1 e^{-st}\cdot t\, dt + \int_1^2 e^{-st}(2-t)\, dt\right]$$

using integration by parts and simplifying ...

$$= \boxed{\frac{1-e^{-s}}{s^2(1-e^{-2s})}}$$