Quiz #14 - HW Quiz

4.3 p 219 #64

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use the Laplace transform to solve

$$y' + y = f(t), \quad y(0) = 0$$

where $f(t) = \begin{cases} 1, & 0 \le t < 1 \\ -1, & t \geq 1 \end{cases}$

<u>SOL</u>: $f(t) = 1 - 1\mathcal{U}(t-1) + (-1)\mathcal{U}(t-1) = 1 - 2\mathcal{U}(t-1)$

$$\mathcal{L}\{y' + y\} = \mathcal{L}\{f(t)\}$$

$$\mathcal{L}\{y'\} + \mathcal{L}\{y\} = \mathcal{L}\{f(t)\}$$

$$(sY(s) - y(0)) + Y(s) = \mathcal{L}\{1 - 2\mathcal{U}(t-1)\}$$

$$sY(s) - 0 + Y(s) = \mathcal{L}\{1\} - 2\underbrace{\mathcal{L}\{\mathcal{U}(t-1)\}}_{\text{using 10a}}$$

$$sY(s) + Y(s) = \frac{1}{s} - 2e^{-s}\mathcal{L}\{1\}$$

$$Y(s)(s+1) = \frac{1}{s} - 2e^{-s}\frac{1}{s}$$

$$Y(s) = \frac{1}{s(s+1)} - \frac{2e^{-s}}{s(s+1)}$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s(s+1)} - \frac{2e^{-s}}{s(s+1)}\right\}$$

$$\frac{1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1} \Rightarrow 1 = A(s+1) + Bs$$

$$1 = (A+B)s + A$$

$$A + B = 0 \Rightarrow A = -B$$

$$\underline{A = 1} \Rightarrow \underline{B = -1}$$

$$y(t) = \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} - \mathcal{L}^{-1}\left\{2e^{-s}\left[\frac{1}{s} - \frac{1}{s+1}\right]\right\}$$

$$= 1 - e^{-t} - 2\mathcal{L}^{-1}\left\{\underbrace{e^{-s}}_{e^{-as}}\underbrace{\left[\frac{1}{s}\right]}_{G(s)}\right\} + 2\mathcal{L}^{-1}\left\{\underbrace{e^{-s}}_{e^{-as}}\underbrace{\left[\frac{1}{s+1}\right]}_{H(s)}\right\}$$

$a = 1$

$G(s) = \frac{1}{s}$

$\Rightarrow g(t) = \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = 1$

$\Rightarrow g(t-1) = 1$

$a = 1$

$H(s) = \frac{1}{s+1}$

$\Rightarrow h(t) = e^{-t}$

$\Rightarrow h(t-1) = e^{-(t-1)}$

by formula #10

$$= 1 - e^{-t} - 2 \cdot 1 \cdot \mathcal{U}(t-1) + 2 \cdot e^{-(t-1)} \cdot \mathcal{U}(t-1)$$

$$= \boxed{1 - e^{-t} - 2\left[1 - e^{-(t-1)}\right]\mathcal{U}(t-1)}$$