Math 371 - Fall 2011 - 11/8/2011 - Quiz #13 **Score:** _____ out of 10.

Name: Key Seat:

Show all work clearly and in order. Please box your answers.

1. Evaluate the following (using any correct method).

(a) $\mathscr{L}\{e^{2t}\sin(3t)\}$

SOL 1: formula #14 with $a=2$, $K=3$

$$\mathscr{L}\{e^{2t}\sin(3t)\} = \frac{3}{(s-2)^2+3^2} = \boxed{\frac{3}{(s-2)^2+9}}$$

SOL 2: use formula # 9 with $a=2$, $f(t)=\sin(3t)$

So $\mathscr{L}\{e^{2t}\sin(3t)\} \overset{\#9}{=} \mathscr{L}\{\sin(3t)\}\big|_{s\to s-2} \overset{\#5}{=} \frac{3}{s^2+9}\Big|_{s\to s-2} = \boxed{\frac{3}{(s-2)^2+9}}$

(b) $\mathscr{L}\{(2t+1)\mathscr{U}(t-2)\}$

10a: use 10a here (it is much easier!)

$f(t) = 2t+1$ and $a=2$ so $f(t+a) = f(t+2) = 2(t+2)+1 = 2t+5$ (don't forget parentheses!)

$$= e^{-as}\mathscr{L}\{f(t+a)\}$$
$$= e^{-2s}\mathscr{L}\{2t+5\}$$
$$= e^{-2s}\left(2\cdot\frac{1}{s^2} + 5\cdot\frac{1}{s}\right) = \boxed{e^{-2s}\left(\frac{2}{s^2}+\frac{5}{s}\right)}$$

(c) $\mathscr{L}\{\cos(4t)\mathscr{U}(t-\pi)\}$

10a: use 10a here

$f(t) = \cos(4t)$ and $a=\pi$ so $f(t+a) = f(t+\pi) = \cos(4(t+\pi)) = \cos(4t+4\pi)$

$= \cos(4t)$ (cosine has period 2π -OR- use trig formula)

$$= e^{-as}\mathscr{L}\{f(t+a)\}$$
$$= e^{-\pi s}\mathscr{L}\{\cos(4t)\}$$
$$= e^{-\pi s}\left(\frac{s}{s^2+4^2}\right) = \boxed{e^{-\pi s}\left(\frac{s}{s^2+16}\right)}$$

2. Write $f(t)$ in terms of unit step functions (Heaviside functions) if

$$f(t) = \begin{cases} e^t, & 0 \le t < 4, \\ t, & t \ge 4. \end{cases}$$

$$\boxed{f(t) = e^t - e^t\,\mathscr{U}(t-4) + t\,\mathscr{U}(t-4)}$$

"turn on e^t at $t=0$" — "turn off e^t at $t=4$" — "turn on t at $t=4$"