Math 371 - Fall 2011 - 11/2/2011 - Quiz #12 **Score:** ______ out of 10.

Name: Key Seat:

Show all work clearly and in order. Please box your answers. Due 11/3/2011.

PICK ONE OF THE FOLLOWING (1 or 2):

Please indicate which one you do NOT want me to grade by putting an X through it, otherwise I will grade the first one worked on:

1. (a) Use the definition of the Laplace transform to find $\mathscr{L}\{f(t)\}$ if

$$f(t) = \begin{cases} -2 & 0 \le t < 1, \\ 1 & t \ge 1. \end{cases}$$

$$\mathscr{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt = \int_0^1 e^{-st}(-2)\,dt + \int_1^\infty e^{-st}\,dt = \left[\frac{-2e^{-st}}{-s}\right]_0^1 + \lim_{b\to\infty}\int_1^b e^{-st}\,dt =$$

$$= \frac{2e^{-s}}{s} - \frac{2}{s} + \lim_{b\to\infty}\left[\frac{e^{-st}}{-s}\right]_1^b = \frac{2e^{-s}}{s} - \frac{2}{s} + \lim_{b\to\infty}\left(\frac{e^{-sb}}{-s} - \frac{e^{-s}}{-s}\right) = \frac{2e^{-s}}{s} - \frac{2}{s} + \frac{e^{-s}}{s} \quad (s>0)$$

$$= \boxed{\frac{3e^{-s}-2}{s}} \quad (s>0)$$

(b) $\mathscr{L}^{-1}\left\{\frac{(s+2)^2}{s^3}\right\} = \mathscr{L}^{-1}\left\{\frac{s^2+4s+4}{s^3}\right\} = \mathscr{L}^{-1}\left\{\frac{1}{s} + \frac{4}{s^2} + \frac{4}{s^3}\right\}$

$$= \mathscr{L}^{-1}\left\{\frac{1}{s}\right\} + \mathscr{L}^{-1}\left\{\frac{4}{s^2}\right\} + \mathscr{L}\left\{\frac{4}{s^3}\right\} = \mathscr{L}^{-1}\left\{\frac{1}{s}\right\} + 4\mathscr{L}^{-1}\left\{\frac{1}{s^2}\right\} + 2\mathscr{L}^{-1}\left\{\frac{2}{s^3}\right\}$$

$$= \boxed{1 + 4t + 2t^2}$$

(c) $\mathscr{L}^{-1}\left\{\frac{1}{s^2-3s+2}\right\} = \mathscr{L}^{-1}\left\{\frac{1}{(s-2)(s-1)}\right\} = \mathscr{L}^{-1}\left\{\frac{1}{s-2} - \frac{1}{s-1}\right\} = \mathscr{L}^{-1}\left\{\frac{1}{s-2}\right\} - \mathscr{L}^{-1}\left\{\frac{1}{s-1}\right\} = \boxed{e^{2t} - e^{t}}$

$$\frac{1}{(s-2)(s-1)} = \frac{A}{s-2} + \frac{B}{s-1} = \frac{A(s-1)+B(s-2)}{(s-1)(s-2)}$$

$1 = As - A + Bs - 2B$

$1 = s(A+B) - A - 2B$

$A+B=0 \Rightarrow A=-B$

$-A-2B=1 \Rightarrow B-2B=1$

$-B=1 \Rightarrow B=-1$

$A=1$

2. Use the Laplace transform to solve the following initial-value problem:

$$y' - y = 2\cos(5t), \quad y(0) = 0.$$

$\mathscr{L}\{y'-y\} = \mathscr{L}\{2\cos(5t)\}$

$\mathscr{L}\{y'\} - \mathscr{L}\{y\} = 2\,\mathscr{L}\{\cos(5t)\}$

$(sY(s) - y(0)) - Y(s) = 2\left(\frac{s}{s^2+25}\right)$

$sY(s) - 0 - Y(s) = \frac{2s}{s^2+25}$

$Y(s)(s-1) = \frac{2s}{s^2+25}$

$Y(s) = \frac{2s}{(s-1)(s^2+25)}$

$y(t) = \mathscr{L}^{-1}\{Y(s)\} = \mathscr{L}^{-1}\left\{\frac{2s}{(s-1)(s^2+25)}\right\}$

use partial fractions:

$\frac{2s}{(s-1)(s^2+25)} = \frac{A}{s-1} + \frac{Bs+C}{s^2+25}$

$2s = A(s^2+25) + (Bs+C)(s-1)$

$2s = As^2 + 25A + Bs^2 - Bs + Cs - C$

$2s = (A+B)s^2 + (C-B)s - C + 25A$

$A+B=0 \Rightarrow A=-B$

$C-B=2 \Rightarrow B=C-2$

$-C+25A=0 \Rightarrow C=25A$

so $C = 25A = -25B = -25(C-2) = -25C+50$

$26C = 50$

$C = \frac{50}{26} = \frac{25}{13}$, $B = \frac{25}{13} - 2$

$= \frac{25}{13} - \frac{26}{13}$

$= \frac{-1}{13}$

$$A = -B = \frac{1}{13}$$

So

$$y(t) = \mathcal{L}^{-1}\left\{\frac{25}{(s-1)(s^2+25)}\right\} = \mathcal{L}^{-1}\left\{\frac{1/13}{s-1} + \frac{(-1/13)s + (25/13)}{s^2+25}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{13}\cdot\frac{1}{s-1} - \frac{1}{13}\cdot\frac{s}{s^2+25} + \frac{25}{13}\cdot\frac{1}{s^2+25}\right\}$$

$$= \frac{1}{13}\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} - \frac{1}{13}\mathcal{L}^{-1}\left\{\frac{s}{s^2+25}\right\} + \frac{5}{13}\mathcal{L}^{-1}\left\{\frac{5}{s^2+25}\right\}$$

$$= \boxed{\frac{1}{13}e^{t} - \frac{1}{13}\cos(5t) + \frac{5}{13}\sin(5t)}$$