 Score: _____ out of 10.

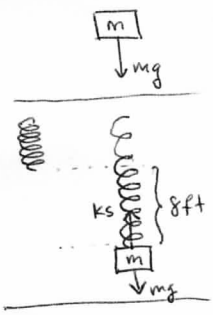
Name: key Seat:

Show all work clearly and in order. Please box your answers. Due 10/27/2011.

1. An 8-pound weight stretches a spring 8 feet. Assume a damping force numerically equal to the instantaneous velocity acts on the system. Suppose the weight is released at the equilibrium position with downward velocity of 1 ft/s.

(a) Find the equation of motion.

pictures:



To find the mass m: Weight $= mg$

$8\text{lb} = m(32\text{ft/s}^2) \Rightarrow m = \frac{8}{32} = \frac{1}{4}$ slug

To find the spring constant k: $mg = ks$

$8\text{lb} = k(8\text{ft}) \Rightarrow k = 1\ \text{lb/ft}$

To find damping constant β: The second sentence tells us $\beta = 1$ so that $\beta\frac{dx}{dt} = \frac{dx}{dt}$

Initial conditions: $x(0) = 0$ and $x'(0) = +1$

$$m\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + kx = 0$$

$$\frac{1}{4}\frac{d^2x}{dt^2} + \frac{dx}{dt} + x = 0$$

$$x'' + 4x' + 4x = 0$$

→ 2nd order linear, homogeneous w/ constant coef.

$m^2 + 4m + 4 = 0$

$(m+2)(m+2) = 0$

$m = -2 \mid m = -2$

$x(t) = C_1e^{-2t} + C_2te^{-2t}$

$x(0) = C_1 + 0 = 0 \Rightarrow C_1 = 0$

$x(t) = C_2te^{-2t}$

(b) What is the velocity of the weight after 1 second (approximate to the nearest hundredth)?

$x'(t) = -2te^{-2t} + e^{-2t}$

$x'(1) = -2e^{-2} + e^{-2} = -e^{-2} \approx \boxed{-0.14}$ ft/s

So

$x'(t) = C_2 + (-2e^{-2t}) + C_2e^{-2t}$

$x'(0) = 1 = 0 + C_2(1)$

$\Rightarrow C_2 = 1$

So $\boxed{x(t) = te^{-2t}}$