4.3

24 use the Laplace transform to solve :

$$y'' - 4y' + 4y = t^3 e^{2t}$$

$$y(0) = 0$$

$$y'(0) = 0$$

<u>SOL</u> :

$$\mathcal{L}\{y'' - 4y' + 4y\} = \mathcal{L}\{t^3 e^{2t}\}$$

$$\mathcal{L}\{y''\} - 4\mathcal{L}\{y'\} + 4\mathcal{L}\{y\} = \mathcal{L}\{t^3 e^{2t}\}$$ by formula #9

$$\left(s^2 Y(s) - sy(0) - y'(0)\right) - 4\left(sY(s) - y(0)\right) + 4Y(s) = \mathcal{L}\{t^3\}\Big|_{s \to s-2}$$

$$s^2 Y(s) - 0 - 0 - 4\left(sY(s) - 0\right) + 4Y(s) = \frac{3!}{s^4}\Big|_{s \to s-2}$$

$$s^2 Y(s) - 4sY(s) + 4Y(s) = \frac{6}{(s-2)^4}$$ (you can also use formula #13 to get here)

$$Y(s)\left(s^2 - 4s + 4\right) = \frac{6}{(s-2)^4}$$

$$Y(s) = \frac{6}{(s^2 - 4s + 4)(s-2)^4} = \frac{6}{(s-2)^2 (s-2)^4} = \frac{6}{(s-2)^6}$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{6}{(s-2)^6}\right\} = \mathcal{L}^{-1}\left\{\frac{6}{s^6}\Big|_{s \to s-2}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{6}{5!} \cdot \frac{5!}{s^6}\Big|_{s \to s-2}\right\}$$

$\downarrow$

$$= \frac{6}{5!}\,\mathcal{L}^{-1}\left\{ \left.\frac{5!}{s^6}\right|_{s \to s-2} \right\}$$

$$= \frac{\cancel{6}}{5\cdot 4\cdot 3\cdot \cancel{2\cdot 1}}\,\mathcal{L}^{-1}\left\{ \left.\frac{5!}{s^6}\right|_{\underbrace{s \to s-2}_{a=2}} \right\}$$

$$= \boxed{\frac{1}{20}\, e^{2t}\, t^5} \leftarrow \text{(by formula \# 9)}$$