Math 371 - Midterm Review Questions

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Problem 1

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Seperable (Section 2.2)

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Any singular solutions?

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Any singular solutions? NO! $e^{2y} \neq 0$ for all $y \in \mathbb{R}$.

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Any singular solutions? YES! $y = 0$.

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Determine whether this DE is exact. If it is solve it,

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(5) Solution:

$$f(x,y) = C.$$

Hence,

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2. DE becomes:

$$\begin{aligned}
uxdx - (2x + 2ux)(udx + xdu) &= 0 \\
uxdx - (2xudx + 2x^2du + 2u^2xdx + 2ux^2du) &= 0 \\
uxdx - 2xudx - 2x^2du - 2u^2xdx - 2ux^2du &= 0 \\
(ux - 2xu - 2u^2x)dx + (-2x^2 - 2ux^2)du &= 0 \\
x(-u - 2u^2)dx + x^2(-2 - 2u)du &= 0 \\
x(-u - 2u^2)dx &= -x^2(-2 - 2u)du \\
\frac{x}{x^2}dx &= \frac{-(-2 - 2u)}{-u - 2u^2}du \\
\frac{x}{x^2}dx &= \frac{2(1 + u)}{-u(1 + 2u)}du
\end{aligned}$$

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\frac{y}{y^2}dy &= \frac{1}{u+2}du \\
\ln|y| + C &= \ln|u + 2|
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y(-u - 2) dy + y^2 du &= 0 \\
y(-u - 2) dy &= -y^2 du \\
\frac{y}{y^2} dy &= \frac{1}{u + 2} du \\
\ln|y| + C &= \ln|u + 2| \\
\ln|y| + C &= \ln|x/y + 2|
\end{aligned}$$

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$$x\left(-\frac{1}{u^2}\frac{du}{dx}\right) + \frac{1}{u} = x^2\left(\frac{1}{u}\right)^2$$

$$\frac{du}{dx} - \frac{1}{x}u = -x$$

Now this is 1st Order Linear. Solution:

$$y = 1/(-x^2 + Cx).$$

Problem 7 - Reduction of Order (Section 3.2)

$y_1 = e^{x/3}$ is a solution of $6y'' + y' - y = 0$. Use reduction of order to find a second independent solution y_2.

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$$y'' - 10y' + 25y = 0$$

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2nd Order Linear Homogeneous with Constant Coef. (Section 3.3)

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(1) FIND y_c, the general solution to the associated homogeneous DE:

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Substituting into $y'' - 10y' + 25y = e^x$:

$$Ae^x - 10Ae^x + 25Ae^x = e^x$$
$$(A - 10A + 25A)e^x = e^x$$

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$$
\begin{aligned}
y_p &= Ae^x \\
y_p' &= Ae^x \\
y_p'' &= Ae^x
\end{aligned}
$$

Substituting into $y'' - 10y' + 25y = e^x$:

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So $16A = 1 \implies A = 1/16$.

Hence,

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Problem 11

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2nd Order Linear Nonomogeneous with Constant Coef. (Section 3.4/3.5)

(1) FIND y_c, the general solution to the associated homogeneous DE:

$$y'' - 10y' + 25y = 0.$$

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$1m^2 - 10m + 25 = 0 \implies (m-5)(m-5) = 0 \implies m = 5, m = 5.$

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2. $y_1 = e^{5x}$, $y_2 = xe^{5x}$, $f(x) = e^{5x}$ and the Wronskian of y_1 and y_2 is $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{5x} & xe^{5x} \\ 5e^{5x} & 5xe^{5x} + e^{5x} \end{vmatrix} = e^{10x}$.
3. $u_1' = \dfrac{-y_2 f(x)}{W} = \dfrac{-xe^{5x}e^{5x}}{e^{10x}} = -x$ so $u_1 = -x^2/2$.

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$$y = y_c + y_p = C_1 e^{5x} + C_2 xe^{5x} + \frac{1}{2}x^2 e^{5x}.$$

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2nd Order Homogeneous Cauchy-Euler Equation (Section 3.6)

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- General Solution:

$$y = C_1x^4 + C_2x^{-1}$$