see quiz 11 for another example

(3.8) # 29 Lecture on 12/01

SOL: Find m: weigh $= mg = 16\text{ lbs} = m\,(32\text{ft}/s^2) \Rightarrow m = 1/2$ slug.

Find k: $mg = ks \Rightarrow 16\text{ lbs} = k\,(8/3\text{ ft}) \Rightarrow k = \frac{48}{8} = 6$

Find β: $\beta = 1/2$

Find $f(t)$: $f(t) = 10\cos(3t)$

Initial Conditions: $x(0) = 2$ AND $x'(0) = 0$. ← NOTE typo in the solutions.

so by Equation (24) on p149:

$$\frac{1}{2}\frac{d^2x}{dt^2} = -6x - \frac{1}{2}\frac{dx}{dt} + 10\cos(3t)$$

Solve this I.V.P.:
$$\begin{cases} x'' + x' + 12x = 20\cos(3t) \\ x(0) = 2 \\ x'(0) = 0 \end{cases}$$

see notes from class for details.

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see quiz 12 for another example

(4.1) # 3. Find $\mathcal{L}\{f(t)\}$ if $f(t) = \begin{cases} t & 0 \le t < 1 \\ 1 & t \ge 1 \end{cases}$

SOL: Method 1 (by definition)

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt = \int_0^1 e^{-st}(t)\,dt + \int_1^\infty e^{-st}(1)\,dt$$

$$\begin{array}{l|l} u = t & dv = e^{-st} \\ du = dt & v = \frac{e^{-st}}{-s} \end{array}$$

$$= \left[\frac{te^{-st}}{-s}\right]_0^1 - \int_0^1 \frac{e^{-st}}{-s}\,dt + \lim_{b\to\infty}\int_1^b e^{-st}\,dt$$

$$= \left[\frac{e^{-s}}{-s} - 0\right] - \left[\frac{e^{-st}}{s^2}\right]_0^1 + \lim_{b\to\infty}\left[\frac{e^{-st}}{-s}\right]_1^b$$

$$= \frac{e^{-s}}{-s} - \left[\frac{e^{-s}}{s^2} - \frac{1}{s^2}\right] + \lim_{b\to\infty}\left[\cancel{\frac{e^{-sb}}{-s}}^{\,0 \text{ if } s>0.} - \frac{e^{-s}}{-s}\right]$$

$$= \boxed{\frac{1}{s^2} - \frac{e^{-s}}{s^2}}$$

Method 2 (via Heaviside functions)

$$f(t) = t - t\,\mathcal{U}(t-1) + 1\cdot\mathcal{U}(t-1)$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{t - t\,\mathcal{U}(t-1) + \mathcal{U}(t-1)\}$$

$$= \mathcal{L}\{t\} - \mathcal{L}\{\underbrace{t\,\mathcal{U}(t-1)}_{a=1}\} + \mathcal{L}\{\mathcal{U}(t-\underset{a=1}{1})\}$$

$f(t) = t$ $f(t) = 1$

Formula #: #2 #10a #10a

$$= \frac{1}{s^2} - e^{-s}\mathcal{L}\{t+1\} + e^{-s}\mathcal{L}\{1\}$$

$$= \frac{1}{s^2} - e^{-s}\left(\frac{1}{s^2} + \frac{1}{s}\right) + e^{-s}\left(\frac{1}{s}\right)$$

$$= \boxed{\frac{1}{s^2} - \frac{e^{-s}}{s^2}}$$

4.2 #38. Use Laplace transform to solve the given IVP: $\begin{cases} y'' + 9y = e^{t} \\ y(0) = 0 \\ y'(0) = 0 \end{cases}$

SOL:

$$\mathcal{L}\{y'' + 9y\} = \mathcal{L}\{e^{t}\}$$
$$\mathcal{L}\{y''\} + 9\mathcal{L}\{y\} = \mathcal{L}\{e^{t}\}$$
$$\left[s^2 Y(s) - s\,y(0) - y'(0)\right] + 9Y(s) = \frac{1}{s-1}$$
$$s^2 Y(s) - 0 - 0 + 9Y(s) = \frac{1}{s-1}$$
$$Y(s)(s^2+9) = \frac{1}{s-1}$$
$$Y(s) = \frac{1}{(s-1)(s^2+9)} = \frac{A}{s-1} + \frac{Bs+C}{s^2+9}$$

$$1 = A(s^2+9) + (Bs+C)(s-1)$$
$$1 = As^2 + 9A + Bs^2 + Cs - Bs - C$$
$$1 = (A+B)s^2 + (C-B)s + 9A - C$$

$$A + B = 0 \Rightarrow A = -B$$
$$C - B = 0 \Rightarrow C = B = -A$$
$$9A - C = 1 \Rightarrow 9A + A = 1$$
$$10A = 1$$
$$A = 1/10$$
$$B = -1/10$$
$$C = -1/10$$

so

$$Y(s) = +\frac{1}{10}\cdot\frac{1}{s-1} + \frac{-\frac{1}{10}s - \frac{1}{10}}{s^2+9}$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{10}\left(\frac{1}{s-1}\right) - \frac{1}{10}\left(\frac{s}{s^2+9}\right) - \frac{1}{10}\left(\frac{1}{s^2+9}\right)\right\}$$
$$= \frac{1}{10}\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} - \frac{1}{10}\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\} - \frac{1}{10}\mathcal{L}^{-1}\left\{\frac{1}{s^2+9}\right\}$$
$$= \frac{1}{10}(e^{t}) - \frac{1}{10}\cos(3t) - \frac{1}{10}\mathcal{L}^{-1}\left\{\frac{1}{3}\cdot\frac{3}{s^2+9}\right\}$$
$$= \boxed{\frac{1}{10}e^{t} - \frac{1}{10}\cos(3t) - \frac{1}{30}\sin(3t)}$$

see "the quiz you never took" for another example.

(4.3) # 22. use the Laplace transform to solve the IVP: $\begin{cases} y' - y = 1 + te^t \\ y(0) = 0. \end{cases}$

SOL:

$$\mathcal{L}\{y' - y\} = \mathcal{L}\{1 + te^t\}$$

$$\mathcal{L}\{y'\} - \mathcal{L}\{y\} = \mathcal{L}\{1\} + \underbrace{\mathcal{L}\{te^t\}}_{\# 13}$$

$$[sY(s) - y(0)] - Y(s) = \frac{1}{s} + \frac{1!}{(s-1)^{1+1}} = \frac{1}{s} + \frac{1}{(s-1)^2}$$

$$sY(s) - 0 - Y(s) = \frac{1}{s} + \frac{1}{(s-1)^2}$$

$$Y(s)(s-1) = \frac{1}{s} + \frac{1}{(s-1)^2}$$

$$Y(s) = \frac{1}{s(s-1)} + \frac{1}{(s-1)^3}$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s(s-1)}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{(s-1)^3}\right\}$$

$$\frac{1}{s(s-1)} = \frac{A}{s} + \frac{B}{s-1} \Rightarrow 1 = A(s-1) + Bs$$

$$1 = As - A + Bs$$

$$1 = (A+B)s - A$$

$$-A = 1 \Rightarrow A = -1$$

$$A + B = 0 \Rightarrow B = -A = 1$$

$$= \mathcal{L}^{-1}\left\{\frac{-1}{s} + \frac{1}{s-1}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s^3}\Big|_{s \to s-1}\right\}$$

$a = 1$ and $F(s) = \frac{1}{s^3}$

so $f(t) = \mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\}$

$$= \mathcal{L}^{-1}\left\{\frac{1}{2} \cdot \frac{2}{s^3}\right\}$$

$$= \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{2}{s^3}\right\}$$

$$= \frac{1}{2}t^2$$

$$= -\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} + \underbrace{e^t \cdot \frac{1}{2}t^2}_{\# 9}$$

$$= \boxed{-1 + e^t + \frac{1}{2}e^t t^2}$$

See quiz 13 and 14 for more examples.

(4.3) #63. Use the Laplace transform to solve the given IVP: $\begin{cases} y' + y = f(t) \\ y(0) = 0 \\ f(t) = \begin{cases} 0, & 0 \le t < 1 \\ 5, & t \geq 1 \end{cases} \end{cases}$

Sol: First write $f(t)$ in terms of Heaviside functions.

$$f(t) = 0 - 0\,\mathcal{U}(t-1) + 5\cdot \mathcal{U}(t-1)$$
$$= 5\,\mathcal{U}(t-1)$$

$$y' + y = f(t)$$
$$\mathcal{L}\{y' + y\} = \mathcal{L}\{f(t)\}$$
$$\mathcal{L}\{y'\} + \mathcal{L}\{y\} = \mathcal{L}\{5\,\mathcal{U}(t-1)\}$$
$$[sY(s) - y(0)] + Y(s) = 5\mathcal{L}\{\mathcal{U}(t-1)\}$$

#10a $f(t) = 1$ and $a = 1$

$$sY(s) - 0 + Y(s) = 5\cdot e^{-s}\mathcal{L}\{1\}$$
$$Y(s)(s+1) = 5e^{-s}\left(\frac{1}{s}\right)$$

$$Y(s) = \frac{5e^{-s}}{s(s+1)} = 5e^{-s}\cdot \frac{1}{s(s+1)}$$

$$\frac{1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1} \Rightarrow$$

$1 = A(s+1) + Bs$

$1 = As + A + Bs$

$1 = (A+B)s + A$

$A = 1$

$A + B = 0 \Rightarrow B = -A = -1$

$$\frac{1}{s(s+1)} = \frac{1}{s} - \frac{1}{s+1}$$

$$= 5e^{-s}\left[\frac{1}{s} - \frac{1}{s+1}\right]$$

So

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{5e^{-s}\cdot\frac{1}{s}\right\} - \mathcal{L}^{-1}\left\{5e^{-s}\cdot\frac{1}{s+1}\right\}$$

$$= 5\mathcal{L}^{-1}\underbrace{\left\{e^{-s}\cdot\frac{1}{s}\right\}} - 5\mathcal{L}^{-1}\underbrace{\left\{e^{-s}\cdot\frac{1}{s+1}\right\}}$$

use #10: $a = 1$, $F(s) = \frac{1}{s}$, $f(t) = \mathcal{L}^{-1}\{\frac{1}{s}\} = 1$; $a = 1$, $F(s) = \frac{1}{s+1}$, $f(t) = \mathcal{L}^{-1}\{\frac{1}{s+1}\} = e^{-t}$

$$= \boxed{5\cdot 1\cdot \mathcal{U}(t-1) - 5\cdot e^{-(t-1)}\,\mathcal{U}(t-1)}$$

see quiz 15 for more examples.

4.4 # 26 : $\mathcal{L}\left\{ \int_0^t \underbrace{\tau \sin(\tau)}\, d\tau \right\}$

$f(\tau) = \tau \sin(\tau)$

so $f(t) = t \sin(t)$

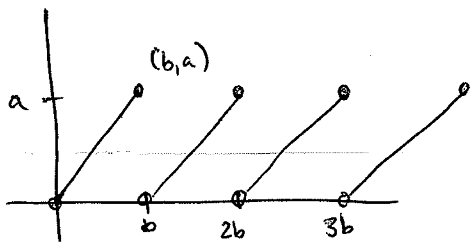
$\overset{*11}{\Longrightarrow}$ $F(s) = (-1)^1 \frac{d}{ds} \mathcal{L}\{\sin(t)\}$

$F(s) = -\frac{d}{ds}\left[\frac{1}{s^2+1}\right] = -\left[\frac{-2s}{(s^2+1)^2}\right]$

by #19

$$\frac{F(s)}{s} = \frac{2\cancel{s}}{\cancel{s}(s^2+1)^2} = \boxed{\frac{2}{(s^2+1)^2}}$$

4.4 #51 Find $\mathcal{L}\{f(t)\}$ if the graph of $f(t)$ is given by



SOL: the period is $T = b$

using thm 4.4.3 on p226 : $\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\, dt$

$$= \frac{1}{1-e^{-bs}} \int_0^b e^{-st} f(t)\, dt$$

what is $f(t)$ on the interval $[0,b]$?

Notice that the line passing through $(0,0)$ and (b,a) is given by

$$y - 0 = \frac{a-0}{b-0}(t - 0)$$

$$y = \frac{a}{b} t$$

Hence, $f(t) = \frac{a}{b} t$ if $0 \le t < b$

AND $f(t+b) = f(t)$ for all t.

Hence

$$\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-bs}} \int_0^b e^{-st} \cdot \frac{a}{b} t\, dt = \boxed{\frac{a}{s}\left[\frac{1}{bs} - \frac{1}{e^{bs}-1}\right]}$$

integration by parts.

see quiz #16 for another example.

4.5 # 4 use the Laplace transform to solve the IVP: $\begin{cases} y'' + 16y = \delta(t-2\pi) \\ y(0)=0 \\ y'(0)=0 \end{cases}$

SOL:

$$\mathcal{L}\{y'' + 16y\} = \mathcal{L}\{\delta(t-2\pi)\}$$

$$\mathcal{L}\{y''\} + 16\mathcal{L}\{y\} = \mathcal{L}\{\delta(t-2\pi)\}$$

$$[s^2Y(s) - sy(0) - y'(0)] + 16Y(s) = e^{-2\pi s}$$

$$s^2Y(s) - 0 - 0 + 16Y(s) = e^{-2\pi s}$$

$$Y(s)(s^2+16) = e^{-2\pi s}$$

$$Y(s) = \frac{e^{-2\pi s}}{s^2+16} = \underbrace{e^{-2\pi s}}_{e^{-as}} \cdot \underbrace{\frac{1}{s^2+16}}_{F(s)}$$

to find $y(t) = \mathcal{L}^{-1}\{Y(s)\}$ using # 10 : $a = 2\pi$ so $f(t) = \mathcal{L}^{-1}\{\frac{1}{s^2+16}\} = \mathcal{L}^{-1}\{\frac{1}{4}\cdot\frac{4}{s^2+16}\}$

$$= \frac{1}{4}\sin(4t)$$

so $y(t) = \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\{e^{-2\pi s}\cdot\frac{1}{s^2+16}\}$

$$= \frac{1}{4}\sin(4(t-2\pi))\,\mathcal{U}(t-2\pi)$$

$$= \frac{1}{4}\sin(4t-8\pi)\,\mathcal{U}(t-2\pi)$$

$$= \boxed{\frac{1}{4}\sin(4t)\,\mathcal{U}(t-2\pi)}$$

see quiz #17 for another example.

5.1 Find two power series solutions of

$$y'' + x^2 y = 0$$

about the ordinary point $x=0$.

SOL: Since $x=0$ is an ordinary point we have a solution of the form:

$$y = \sum_{n=0}^{\infty} c_n x^n$$

$$y' = \sum_{n=0}^{\infty} n c_n x^{n-1} = \sum_{n=1}^{\infty} n c_n x^{n-1}$$

$$y'' = \sum_{n=0}^{\infty} n(n-1) c_n x^{n-2} = \sum_{n=2}^{\infty} n(n-1) c_n x^{n-2}$$

Thus, $y'' + x^2 y = 0$ becomes

$$\sum_{n=2}^{\infty} n(n-1) c_n x^{n-2} + x^2 \sum_{n=0}^{\infty} c_n x^n = 0$$

$$\underbrace{\sum_{n=2}^{\infty} n(n-1) c_n x^{n-2}}_{\text{starts } x^0} + \underbrace{\sum_{n=0}^{\infty} c_n x^{n+2}}_{\text{starts } x^2} = 0$$

Now take out terms so the series start with the same power of x.

$$2\cdot 1 \cdot c_2 x^0 + 3\cdot 2 \cdot c_3 x^1 + \sum_{n=4}^{\infty} n(n-1) c_n x^{n-2} + \sum_{n=0}^{\infty} c_n x^{n+2} = 0$$

"SHIFT n=4 to n=0"

$$2c_2 + 6c_3 x + \underbrace{\sum_{n=0}^{\infty} (n+4)(n+3) c_{n+4} x^{n+2}}_{\text{starts at } x^2\ (n=0)} + \underbrace{\sum_{n=0}^{\infty} c_n x^{n+2}}_{\text{starts at } x^2\ (n=0)} = 0$$

$$2c_2 + 6c_3 x + \sum_{n=0}^{\infty} \left[(n+4)(n+3) c_{n+4} + c_n \right] x^{n+2} = 0$$

By the identity property

$2c_2 = 0$ AND $6c_3 = 0$ AND $(n+4)(n+3)c_{n+4} + c_n = 0$ for $n = 0, 1, 2, 3, \ldots$

$c_2 = 0$ AND $c_3 = 0$ AND

$$c_{n+4} = \frac{-c_n}{(n+4)(n+3)}$$

always solve for higher coef.

set up the table:

$c_0 = ?$
$c_1 = ?$
$c_2 = 0$
$c_3 = 0$

n	$c_{n+4} = \frac{-c_n}{(n+4)(n+3)}$
0	$c_4 = \frac{-c_0}{4 \cdot 3}$
1	$c_5 = \frac{-c_1}{5 \cdot 4}$
2	$c_6 = \frac{-c_2}{6 \cdot 5} = 0$
3	$c_7 = \frac{-c_3}{7 \cdot 6} = 0$
4	$c_8 = \frac{-c_4}{8 \cdot 7} = \frac{c_0}{8 \cdot 7 \cdot 4 \cdot 3}$
5	$c_9 = \frac{-c_5}{9 \cdot 8} = \frac{c_1}{9 \cdot 8 \cdot 5 \cdot 4}$
	$\vdots$

Hence

$$y = \sum_{n=0}^{\infty} c_n x^n$$

$$= c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + \cdots$$

$$= c_0 + c_1 x + 0x^2 + 0x^3 + \frac{-c_0}{4 \cdot 3} x^4 + \frac{-c_1}{5 \cdot 4} x^5 + 0x^6 + 0x^7 + \frac{c_0}{8 \cdot 7 \cdot 4 \cdot 3} x^8 + \frac{c_1}{9 \cdot 8 \cdot 5 \cdot 4} x^9 + \cdots$$

$$= \left(c_0 - \frac{c_0}{4 \cdot 3} x^4 + \frac{c_0}{8 \cdot 7 \cdot 4 \cdot 3} x^8 + \cdots \right) + \left(c_1 x + \frac{-c_1}{5 \cdot 4} x^5 + \frac{c_1}{9 \cdot 8 \cdot 5 \cdot 4} x^9 + \cdots \right)$$

$$= c_0 \underbrace{\left(1 - \frac{1}{4 \cdot 3} x^4 + \frac{1}{8 \cdot 7 \cdot 4 \cdot 3} x^8 + \cdots \right)}_{y_1} + c_1 \underbrace{\left(x - \frac{1}{5 \cdot 4} x^5 + \frac{1}{9 \cdot 8 \cdot 5 \cdot 4} x^9 + \cdots \right)}_{y_2}$$

two solutions

$$\boxed{\begin{aligned} y_1 &= 1 - \frac{1}{4 \cdot 3} x^4 + \frac{1}{8 \cdot 7 \cdot 4 \cdot 3} x^8 + \cdots \\ y_2 &= x - \frac{1}{5 \cdot 4} x^5 + \frac{1}{9 \cdot 8 \cdot 5 \cdot 4} x^9 + \cdots \end{aligned}}$$

(5.2) # 16

$x=0$ is a regular singular point of

$$2xy'' + 5y' + xy = 0$$

use the method of Frobenius to find two linearly independent series solutions about $x=0$.

Sol :

we want solutions of the form

$$y = \sum_{n=0}^{\infty} c_n x^{n+r}$$

substituting

$$y' = \sum_{n=0}^{\infty} (n+r) c_n x^{n+r-1}$$

$$y'' = \sum_{n=0}^{\infty} (n+r)(n+r-1) c_n x^{n+r-2}$$

$$2xy'' + 5y' + xy = 0$$

$$2x \sum_{n=0}^{\infty} (n+r)(n+r-1) c_n x^{n+r-2} + 5 \sum_{n=0}^{\infty} (n+r) c_n x^{n+r-1} + x \sum_{n=0} c_n x^{n+r} = 0$$

$$\underbrace{\sum_{n=0}^{\infty} 2(n+r)(n+r-1) c_n x^{n+r-1}}_{\text{starts } x^{r-1}} + \underbrace{\sum_{n=0}^{\infty} 5(n+r) c_n x^{n+r-1}}_{\text{starts } x^{r-1}} + \underbrace{\sum_{n=0}^{\infty} c_n x^{n+r+1}}_{\text{starts } x^{r+1}} = 0$$

$$2r(r-1)c_0 x^{r-1} + 2(1+r)r\, c_1 x^r + 5r c_0 x^{r-1} + 5(r+1) c_1 x^r +$$
$$\sum_{n=2}^{\infty} 2(n+r)(n+r-1) c_n x^{n+r-1} + \sum_{n=2}^{\infty} 5(n+r) c_n x^{n+r-1} + \sum_{n=0}^{\infty} c_n x^{n+r+1} = 0$$

$$(2r^2 - 2r + 5r) c_0 x^{r-1} + (2r + r^2 + 5r + 5) c_1 x^r +$$
$$\sum_{n=2}^{\infty} 2(n+r)(n+r-1) c_n x^{n+r-1} + \sum_{n=2}^{\infty} 5(n+r) c_n x^{n+r-1} + \sum_{n=2}^{\infty} c_{n-2} x^{n+r-1} = 0$$

$$(2r^2+3r) c_0 x^{r-1} + (r^2 + 7r + 5) c_1 x^r + \sum_{n=2}^{\infty} \left[2(n+r)(n+r-1) c_n + 5(n+r) c_n + c_{n-2} \right] x^{n+r-1} = 0$$

$$(2r^2 + 3r) c_0 = 0 \quad \underline{\text{AND}} \quad (r^2 + 7r + 5) c_1 = 0 \quad \underline{\text{AND}} \quad 2(n+r)(n+r-1)c_n + 5(n+r)c_n + c_{n-2} = 0$$

for $n = 2, 3, 4, \ldots$

$$2r^2 + 3r = 0$$
$$r(2r+3) = 0$$
$$r = 0 \quad \underline{\text{or}} \quad r = -3/2$$

For $r=0$

$c_1 = 0$

and the recurrence relation is

$$C_n = \frac{-C_{n-2}}{2(n+0)(n+0-1)+5(n+0)}$$

$$C_n = \frac{-C_{n-2}}{n(2n+3)}, \quad n=2,3,4,\ldots$$

$C_0 = ?$
$C_1 = 0$

n	$C_n = \frac{-C_{n-2}}{n(2n+3)}$
2	$-\frac{1}{14}C_0$
3	0
4	$\frac{1}{616}C_0$

solution is of the form

$$y = \sum_{n=0}^{\infty} C_n x^{n+r}$$

$$= x^r \sum_{n=0}^{\infty} C_n x^n$$

for $r=0$

$$y = x^0\left(\sum_{n=0}^{\infty} C_n x^n\right)$$

$$= x^0(C_0 + C_1x + C_2x^2 + C_3x^3 + \cdots)$$

$$= 1\left(C_0 + 0x + -\frac{1}{14}C_0x^2 + 0 + \frac{1}{616}C_0x^4 + \cdots\right)$$

$$= C_0 - \frac{1}{14}C_0x^2 + \frac{1}{616}C_0x^4 + \cdots$$

$$= C_0\left(1 - \frac{1}{14}x^2 + \frac{1}{616}x^4 + \cdots\right)$$

first solution

For $r = -3/2$

$C_1 = 0$

reccurrence relation is

$$C_n = -\frac{C_{n-2}}{n(2n-3)}, \quad n=2,3,4\ldots$$

$C_0 = ?$
$C_1 = 0$

n	$C_n = \frac{-C_{n-2}}{n(2n-3)}$
2	$= \frac{-C_0}{2(1)} = \frac{-C_0}{2}$
3	0
4	$\frac{-C_2}{4\cdot 5} = \frac{C_0}{4\cdot5\cdot2} = \frac{C_0}{40}$

for $r = -3/2$

$$y = x^{-3/2}\sum_{n=0}^{\infty} C_n x^n$$

$$= x^{-3/2}(C_0 + C_1x + C_2x^2 + \cdots)$$

$$= x^{-3/2}\left(C_0 + 0 + -\frac{1}{2}C_0x^2 + 0 + \frac{C_0}{40}x^4 + \cdots\right)$$

$$= C_0 x^{-3/2}\left(1 - \frac{1}{2}x^2 + \frac{1}{40}x^4 + \cdots\right)$$

second solution.