

Name: _____

Show all work clearly and in order. Please box your answers. 10 minutes.

1. Solve the following differential equation:

$$y'' - 4y' + 3y = 2e^x$$

Step (i): Solve for y_c :

$$y'' - 4y' + 3y = 0$$

$$m^2 - 4m + 3 = 0$$

$$(m-3)(m-1) = 0$$

$$m=3 \mid m=1$$

$$y_c = c_1 e^{3x} + c_2 e^x$$

Step (ii): solve for y_p :

Form for y_p looking at $g(x) = 2e^x$:

$$y_p = Ae^x$$

Adjust looking at y_c since e^x is part of y_c :

$$y_p = Axe^x$$

Now

$$y_p' = Axe^x + Ae^x$$

$$y_p'' = Axe^x + Ae^x + Ae^x = Axe^x + 2Ae^x$$

$$y'' - 4y' + 3y = 2e^x \Rightarrow \cancel{Axe^x} + 2Ae^x - 4\cancel{Axe^x} - 4Ae^x + 3\cancel{Axe^x} = 2e^x$$

$$-2Ae^x = 2e^x$$

$$-2A = 2$$

$$A = -1$$

$$y_p = -xe^x$$

$$y = y_c + y_p \rightarrow$$

General Solution: $y = c_1 e^{3x} + c_2 e^x - xe^x$

2. Using the method of undetermined coefficients write the FORM for the particular solution (y_p) using the given value for $g(x)$ and the general solution of the associated homogeneous equation (y_c). Do NOT solve for the unknown constants, just write the form.

(a) $g(x) = 9e^{2x}$ and $y_c = C_1 e^{-x} + C_2 x e^{-x}$. so

Form of y_p :

$$Ae^{2x}$$

(b) $g(x) = 9e^{2x}$ and $y_c = C_1 e^{2x} + C_2 x e^{2x}$. so

Form of y_p :

$$Ax^2 e^{2x}$$

(c) $g(x) = 3\cos(2x)$ and $y_c = C_1 \sin(2x) + C_2 \cos(2x)$. so

Form of y_p :

$$Ax \sin(2x) + Bx \cos(2x)$$