Math 271 - Quiz #7 **Score:** _____ out of 10.

Name: key

Show all work clearly and in order. Please box your answers. 10 minutes.

1. The function $y_1 = e^{2x}$ is a solution to $y'' - 4y' + 4y = 0$. Use the reduction of order equation formula to find a second solution $y_2(x)$. (NOTE: you do not need to vertify that y_1 is a solution, just find y_2.)

Standard Form: ✓ $y'' - 4y' + 4y = 0$

$P(x) = -4$

$$y_2 = y_1 \int \frac{e^{-\int P(x)dx}}{(y_1)^2} dx = e^{2x} \int \frac{e^{-\int -4\,dx}}{(e^{2x})^2}$$

$$= e^{2x} \int \frac{e^{+4x}}{e^{4x}} dx$$

$$= e^{2x} \int 1\, dx$$

$$= e^{2x} x$$

$$= \boxed{x e^{2x}}$$

2. Find the general solution to the following:

(a) $y'' - 36y = 0$

$m^2 - 36 = 0$

$(m-6)(m+6) = 0$

$m = 6 \quad | \quad m = -6$

$$y = C_1 e^{6x} + C_2 e^{-6x}$$

(b) $y''' + 2y'' + y' = 0$

$m^3 + 2m^2 + m = 0$

$m(m^2 + 2m + 1) = 0$

$m(m+1)(m+1) = 0$

$m = 0 \quad | \quad m = -1 \quad | \quad m = -1$

$y = C_1 e^{0x} + C_2 e^{-1x} + C_3 x e^{-1x}$

$$y = C_1 + C_2 e^{-x} + C_3 x e^{-x}$$

(c) $y'' + 9y = 0$

$m^2 + 9 = 0$

$m^2 = -9$

$m = \pm\sqrt{-9} = \pm 3i$

$\alpha = 0, \quad \beta = 3$

$y = e^{0x}[C_1 \cos(3x) + C_2 \sin(3x)]$

$$y = C_1 \cos(3x) + C_2 \sin(3x)$$