Math 271 - Quiz #6 **Score:** ______ out of 10.

Name: Key

Show all work clearly and in order. Please box your answers. 10 minutes.

1. Complete all of the following parts. **You may not use the auxiliary/characteristic equation method!**

(a) Verify that $y_1 = e^{-x}$ and $y_2 = e^{2x}$ form a fundamental set of solutions of $y'' - y' - 2y = 0$ on $(-\infty, \infty)$.

(i) verify y_1 is a solution:

$$\left.\begin{aligned} y_1 &= e^{-x} \\ y_1' &= -e^{-x} \\ y_1'' &= e^{-x} \end{aligned}\right\} \Rightarrow y'' - y' - 2y = e^{-x} - (-e^{-x}) - 2e^{-x} = 2e^{-x} - 2e^{-x} = 0 \checkmark$$

(ii) verify y_2 is a solution:

$$\left.\begin{aligned} y_2 &= e^{2x} \\ y_2' &= 2e^{2x} \\ y_2'' &= 4e^{2x} \end{aligned}\right\} \Rightarrow y'' - y' - 2y = 4e^{2x} - 2e^{2x} - 2e^{2x} = 0 \checkmark$$

(iii) verify y_1 and y_2 are lin. indep:

$$W(e^{-x}, e^{2x}) = \begin{vmatrix} e^{-x} & e^{2x} \\ -e^{-x} & 2e^{2x} \end{vmatrix} = 2e^{2x}e^{-x} - (-e^{-x})(e^{2x}) = 2e^{x} + e^{x} = 3e^{x} \neq 0. \checkmark$$

(b) Verify that $y_p = \sin(2x)$ forms a particaular solution of $y'' - y' - 2y = -6\sin(2x) - 2\cos(2x)$.

$$y_p' = 2\cos(2x)$$
$$y_p'' = -4\sin(2x)$$
$$y'' - y' - 2y = -4\sin(2x) - 2\cos(2x) - 2\sin(2x) = -6\sin(2x) - 2\cos(2x) \checkmark$$

(c) Use (a) and (b) to write the general solution of $y'' - y' - 2y = -6\sin(2x) - 2\cos(2x)$.

General Solution: $\boxed{y = c_1 e^{-x} + c_2 e^{2x} + \sin(2x)}$