Math 271 - Quiz #4 Score: ______ out of 10.

Name: key

PICK ONE OF THE FOLLOWING:

Please indicate which problem you do NOT want me to grade by putting an X through it, otherwise I will grade the first problem worked on:

Show all work clearly and in order. Please box your answers. 10 minutes.

1. (a) Solve the following differential equation by using an appropriate substitution:

$$(x+y)dx + xdy = 0.$$

SOL: This DE is homogeneous of degree so we can use a substitution of $u = \frac{x}{y}$ or $u = \frac{y}{x}$

$u = \frac{y}{x} \Rightarrow y = ux$

$dy = u\,dx + x\,du$

so,

$$(x+y)dx + x\,dy = 0$$
$$(x+ux)dx + x(u\,dx + x\,du) = 0$$
$$x(1+u)dx + xu\,dx + x^2\,du = 0$$
$$x(1+u)dx + xu\,dx = -x^2\,du$$
$$[x(1+u) + xu]\,dx = -x^2\,du$$
$$x[(1+u)+u]\,dx = -x^2\,du$$
$$x(1+2u)\,dx = -x^2\,du$$
$$\frac{(\cancel{x})dx}{+x^2} = \frac{-1}{1+2u}\,du$$
$$\int \frac{1}{x}\,dx = \int \frac{-1}{1+2u}\,du \quad \rightsquigarrow \quad t = 1+2u \Rightarrow \frac{dt}{du} = 2 \Rightarrow du = \frac{dt}{2}$$
$$\ln|x| = \int \frac{-1}{t}\,\frac{dt}{2} = -\frac{1}{2}\ln|t| + C$$
$$\ln|x| = -\frac{1}{2}\ln|1+2u| + C$$
$$\ln|x| = -\frac{1}{2}\ln\left|1+2\frac{y}{x}\right| + C$$

Implicit (or Explicit) Solution: $\boxed{\ln|x| = -\frac{1}{2}\ln|1+2(y/x)| + C}$

2. Solve the following differential equation by using an appropriate substitution:

$$x\frac{dy}{dx} + y = x^4y^2.$$

$$\frac{dy}{dx} + \frac{1}{x}y = x^3y^2$$

This is a Bernoulli Equation. with $n=2$.

Subsititute: $u = y^{1-n} = y^{1-2} = y^{-1} \Rightarrow y = \frac{1}{u} = u^{-1}$

$$\frac{dy}{dx} = -u^{-2}\cdot\frac{du}{dx}$$

$\frac{dy}{dx} + \frac{1}{x}y = x^3y^2$ becomes

$$\left(-u^{-2}\frac{du}{dx}\right) + \left(\frac{1}{x}\right)\left(\frac{1}{u}\right) = x^3\left(\frac{1}{u}\right)^2$$

$$\frac{du}{dx} + (-u^2)\left(\frac{1}{x}\right)\left(\frac{1}{u}\right) = x^3\left(\frac{1}{u^2}\right)(-u^2)$$

$\frac{du}{dx} - \frac{1}{x}u = -x^3$ ← 1st order linear (good!)

Standard Form: ✓

Integrating Factor: $e^{\int(-\frac{1}{x})dx} = e^{-\ln|x|} = e^{\ln|x|^{-1}} = |x|^{-1} = x^{-1}$, if $x>0$

$= \frac{1}{x}$, if $x>0$

Multiply: $\frac{1}{x}\left[\frac{du}{dx} - \frac{1}{x}u\right] = \left(\frac{1}{x}\right)(-x^3)$

$$\frac{d}{dx}\left[\frac{1}{x}\cdot u\right] = -x^2$$

Integrate: $\frac{1}{x}\cdot u = \int -x^2\,dx$

$$\frac{1}{x}u = -\frac{x^3}{3} + C$$

$$u = -\frac{x^4}{3} + Cx$$

resubstitute: $\frac{1}{y} = -\frac{x^4}{3} + Cx$

Implicit (or Explicit) Solution: $\boxed{\frac{1}{y} = -\frac{x^4}{3} + Cx}$