

Name: _____

key

Show all work clearly and in order. Please box your answers. 10 minutes.

1. (a) Find the explicit general solution for

$$x^2 y' + xy = 1.$$

Standard Form: $y' + \frac{x}{x^2} y = \frac{1}{x^2}$

$$y' + \underbrace{\frac{1}{x}}_{P(x)} y = \frac{1}{x^2}$$

IntegratingFactor

$$e^{\int P(x) dx} = e^{\int \frac{1}{x} dx} = e^{\ln|x|} = |x|$$

$$= x, \text{ if } x > 0$$

Multiply: $x [y' + \frac{1}{x} y] = x [\frac{1}{x^2}]$

$$\frac{d}{dx} [xy] = \frac{1}{x}$$

Integrate:

$$xy = \int \frac{1}{x} dx$$

$$xy = \ln|x| + C$$

$$y = \frac{\ln|x|}{x} + \frac{C}{x}$$

Since we assumed $x > 0$, you can even drop the $| \cdot |$ here.

Explicit Solution:

$$y = \frac{\ln(x)}{x} + \frac{C}{x}$$

- (b) Give the largest interval over which the general solution is defined.

 $(0, \infty)$

- (c) Are there any transient terms in the general solution? If yes, what are they?

 yes, $\frac{C}{x}$ AND $\frac{\ln(x)}{x}$

Note: $\lim_{x \rightarrow \infty} \frac{\ln(x)}{x} = \lim_{x \rightarrow \infty} \left(\frac{1}{x}\right) = 0$