Math 271 - Quiz #2 **Score:** ______ out of 10.

Name: Key

Show all work clearly and in order. Please box your answers. 10 minutes.

1. Find a solution to the following initial-value problem:

$$\frac{dy}{dx} = x\sin(x), \qquad y(\pi/2) = 1.$$

$\int dy = \int x \sin(x)\,dx$ $\quad u = x,\ du = 1\,dx \mid dv = \sin(x),\ v = -\cos(x)$

$y = -x\cos(x) - \int(-\cos(x))\,dx$

$y = -x\cos(x) + \int \cos(x)\,dx$

$y = -x\cos(x) + \sin(x) + C$

$y(\pi/2) = 1 = -\frac{\pi}{2}\cos\left(\frac{\pi}{2}\right) + \sin\left(\frac{\pi}{2}\right) + C$

$1 = 0 + 1 + C$

$C = 0$

Implicit/Explicit Solution: $\boxed{y = -x\cos(x) + \sin(x)}$

2. (a) Find the general solution of

$$\frac{dy}{dx} = \frac{e^x(y+1)}{e^{2x}+1}.$$

$\int \frac{dy}{y+1} = \int \frac{e^x}{e^{2x}+1}\,dx$ $\quad u = e^x \Rightarrow \frac{du}{dx} = e^x \Rightarrow dx = \frac{du}{e^x}$

$\ln|y+1| = \int \frac{\cancel{e^x}}{u^2+1}\cdot\frac{du}{\cancel{e^x}}$

$\ln|y+1| = \int \frac{1}{u^2+1}\,du = \tan^{-1}(u) + C$

$\ln|y+1| = \tan^{-1}(e^x) + C$ $\Leftarrow$ Implicit solution

$|y+1| = e^{\tan^{-1}(e^x)+C} = e^{\tan^{-1}(e^x)}e^C = e^{\tan^{-1}(e^x)}D$

$y+1 = Ee^{\tan^{-1}(e^x)}$

$y = Ee^{\tan^{-1}(e^x)} - 1$

Implicit/Explicit Solution: $\boxed{y = Ee^{\tan^{-1}(e^x)} - 1}$