Math 271 - Fall 2012 - Quiz #9 **Score:** _______ out of 10.

Name: Key

Show all work clearly and in order. Please box your answers.

1. Find two power series solutions of

$$y'' + 2y' = 0$$

about the ordinary point $x = 0$. Find the first three nonzero terms of each power series solution.

$$y = \sum_{n=0}^{\infty} c_n x^n$$

$$y' = \sum_{n=0}^{\infty} c_n n x^{n-1} = \sum_{n=1}^{\infty} c_n n x^{n-1}$$

$$y'' = \sum_{n=0}^{\infty} c_n n(n-1) x^{n-2} = \sum_{n=2}^{\infty} c_n n(n-1) x^{n-2}$$

$$y'' + 2y' = 0$$

$$\sum_{n=2}^{\infty} c_n (n)(n-1) x^{n-2} + \sum_{n=1}^{\infty} 2c_n (n) x^{n-1} = 0$$

Stats at x^0 Stats at x^0 In Phase!

we don't need to fix the phase... so:
Shift so both start at $n=1$

$$\sum_{n=1}^{\infty} c_{n+1}(n+1)((n+1)-1) x^{(n+1)-2} + \sum_{n=1}^{\infty} 2c_n (n) x^{n-1} = 0$$

$$\sum_{n=1}^{\infty} c_{n+1}(n+1)(n) x^{n-1} + \sum_{n=1}^{\infty} 2c_n (n) x^{n-1} = 0$$

$$\sum_{n=1}^{\infty} \left[c_{n+1}(n+1)(n) + 2c_n(n) \right] x^{n-1} = 0$$

Hence, $c_{n+1}(n+1)n + 2c_n n = 0$

$$c_{n+1} = \frac{-2n c_n}{n(n+1)}$$

n	$c_{n+1} = \frac{-2n c_n}{n(n+1)}$
1	$c_2 = \frac{-2(1)c_1}{1(2)} = -c_1$
2	$c_3 = \frac{-2(2)c_2}{2(3)} = -\frac{2}{3} c_2 = -\frac{2}{3}(-c_1) = \frac{2}{3} c_1$

$$y = \sum_{n=0}^{\infty} c_n x^n$$

$$y = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots$$

$$y = c_0 + c_1 x + (-c_1) x^2 + \left(\tfrac{2}{3} c_1\right) x^3 + \cdots$$

$$y = c_0 \underbrace{(1)}_{y_1} + c_1 \underbrace{\left(x - x^2 + \tfrac{2}{3} x^3 + \cdots \right)}_{y_2}$$

so

$$\boxed{\begin{array}{l} y_1 = 1 \\ \hline y_2 = x - x^2 + \frac{2}{3} x^3 + \cdots \end{array}}$$