

Name: _____

Key

Show all work clearly and in order. Please box your answers.

1. Shift the following power series to start at $n = 1$

$$(a) \sum_{n=0}^{\infty} n c_{n+1} x^{n+4} = \sum_{n=1}^{\infty} (n-1) C_n x^{n+3}$$

$$(b) \sum_{n=2}^{\infty} n(n-1) c_{n-1} x^n = \sum_{n=1}^{\infty} (n+1)(n) C_n x^{n+1}$$

2. Combine the following two power series to form a single power series

$$\begin{aligned} & \sum_{n=0}^{\infty} n c_{n+1} x^n + \sum_{n=2}^{\infty} n(n-1) c_n x^{n-1} \\ & \quad \begin{array}{c} \uparrow \\ \text{Starts at} \\ x^0 \end{array} \qquad \begin{array}{c} \uparrow \\ \text{Starts at} \\ x^1 \end{array} \\ & = \underbrace{0 c_{0+1} x^0}_{n=0} + \sum_{n=1}^{\infty} n c_{n+1} x^n + \sum_{n=2}^{\infty} n(n-1) c_n x^{n-1} \\ & \quad \begin{array}{c} \uparrow \\ \text{shift to } n=2 \end{array} \\ & = 0 + \sum_{n=2}^{\infty} (n-1) c_n x^{n-1} + \sum_{n=2}^{\infty} n(n-1) c_n x^{n-1} \\ & = \boxed{\sum_{n=2}^{\infty} [(n-1) c_n + n(n-1) c_n] x^{n-1}} \\ & = \sum_{n=2}^{\infty} (n-1)(1+n) c_n x^{n-1} \end{aligned}$$

3. Combine the following two power series to form a single power series

$$\begin{aligned} & \sum_{n=1}^{\infty} 2n c_n x^{n-1} + \sum_{n=0}^{\infty} 6c_n x^{n+1} \\ & \quad \begin{array}{c} \uparrow \\ \text{Starts at} \\ x^0 \end{array} \qquad \begin{array}{c} \uparrow \\ \text{Starts at} \\ x^1 \end{array} \\ & = \underbrace{2 \cdot 1 \cdot c_1 x^0}_{n=1} + \sum_{n=2}^{\infty} 2n c_n x^{n-1} + \sum_{n=0}^{\infty} 6c_n x^{n+1} \\ & \quad \begin{array}{c} \uparrow \\ \text{Shift to } n=0 \end{array} \\ & = 2c_1 + \sum_{n=0}^{\infty} 2(n+2) c_{n+2} x^{n+1} + \sum_{n=0}^{\infty} 6c_n x^{n+1} \\ & = \boxed{2c_1 + \sum_{n=0}^{\infty} [2(n+2) c_{n+2} + 6c_n] x^{n+1}} \end{aligned}$$