Math 271 - Fall 2012 - Quiz #7 **Score:** ______ out of 10.

Name: ______ Key ______

Show all work clearly and in order. Please box your answers.

1. Solve the following differential equations:

(a) $x^2y'' - 6xy' + 12y = 0$

$$am^2 + (b-a)m + c = 0$$
$$1m^2 + (-6-1)m + 12 = 0$$
$$m^2 - 7m + 12 = 0$$
$$(m-4)(m-3) = 0$$
$$m = 4 \mid m = 3$$

General Solution: $\boxed{y = c_1 x^3 + c_2 x^4}$

(b) $x^2y'' - 3xy' + 4y = 0$

$$1m^2 + (-3-1)m + 4 = 0$$
$$m^2 - 4m + 4 = 0$$
$$(m-2)(m-2) = 0$$
$$m = 2 \mid m = 2$$

General Solution: $\boxed{y = c_1 x^2 + c_2 x^2 \ln(x)}$

(c) $4x^2y'' + 5y = 0$

$$4m^2 + (0-4)m + 5 = 0$$
$$4m^2 - 4m + 5 = 0$$
$$m = \frac{4 \pm \sqrt{16 - 4(5)(4)}}{2(4)} = \frac{4 \pm \sqrt{-64}}{8} = \frac{4 \pm 8i}{8} = \frac{1}{2} \pm i \quad \text{so } \alpha = 1/2 \text{ and } \beta = 1$$

General Solution: $\boxed{y = c_1 x^{1/2} \cos(\ln(x)) + c_2 x^{1/2} \sin(\ln(x))}$

2. Solve the following differential equation (please simplify you final answer):

$$x^2y'' - 3xy' + 4y = x^3$$

Step 1: Find y_c for $x^2y'' - 3xy' + 4y = 0$

$$m^2 + (-3-1)m + 4 = 0$$
$$m^2 - 4m + 4 = 0$$
$$(m-2)(m-2) = 0$$
$$m = 2 \mid m = 2$$
$$y_c = \underbrace{c_1 x^2}_{y_1} + c_2 \underbrace{x^2 \ln(x)}_{y_2} \quad \text{(same as (b) above)}$$

Step 2: Find y_p using variation of parameters:

Standard Form: $y'' - \frac{3}{x}y' + \frac{4}{x^2}y = \underbrace{x}_{f(x)}$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} x^2 & x^2\ln(x) \\ 2x & x^2(\frac{1}{x}) + 2x\ln(x) \end{vmatrix} = x^2(x + 2x\ln(x)) - (2x)(x^2\ln(x))$$
$$= x^3 + 2x^3\ln(x) - 2x^3\ln(x)$$
$$= x^3$$

so now find u_1 and u_2:

$$u_1 = \int -\frac{y_2 f(x)}{W}\,dx = \int -\frac{(x^2\ln(x))(x)}{x^3}\,dx$$
$$= -\int \ln(x)\,dx$$

$u = \ln(x)$ $\quad dv = 1$

$du = \frac{1}{x}dx$ $\quad v = x$

$$= -\left[x\ln(x) - \int 1\,dx\right]$$
$$= -[x\ln(x) - x]$$
$$= x - x\ln(x)$$

$$u_2 = \int \frac{y_1 f(x)}{W}\,dx = \int \frac{x^2 \cdot x}{x^3}\,dx = \int 1\,dx = x$$

so $y_p = u_1y_1 + u_2y_2$
$$= (x - x\ln(x))(x^2) + (x)(x^2\ln(x))$$
$$= x^3 - x^3\ln(x) + x^3\ln(x)$$
$$= x^3$$

Hence $y = y_c + y_p$

Simplified General Solution: $\boxed{y = c_1x^2 + c_2x^2\ln(x) + x^3}$