Math 271 - Fall 2012 - Quiz #6 **Score:** ________ out of 10.

Name: key

Show all work clearly and in order. Please box your answers. 10 minutes.

1. Solve the following differential equation:

$$y'' - 3y' + 2y = \underbrace{3e^{2x}}_{g(x)}$$

SOL 1: Method of undetermined coef.

Step 1: Find y_c for $y''-3y'+2y=0$

$m^2-3m+2=0$

$(m-2)(m-1)=0$

$m=2 \mid m=1$

so $y_c = c_1e^x + c_2e^{2x}$

Step 2: Find y_p (using method of undet coef.)

looking at $g(x)$ the form for y_p is:

$y_p = Ae^{2x}$

BUT e^{2x} is part of y_c so we need to change:

$y_p = Axe^{2x}$

so $y_p' = (Ax)(2e^{2x}) + (e^{2x})A = 2Axe^{2x} + Ae^{2x}$

$y_p'' = (2Ax)(2e^{2x}) + (e^{2x})(2A) + (A)(2e^{2x})$

$= 4Axe^{2x} + 4Ae^{2x}$

plug into $y_p''-3y_p'+2y_p = 3e^{2x}$:

$[4Axe^{2x}+4Ae^{2x}] - 3[2Axe^{2x}+Ae^{2x}] + 2[Axe^{2x}] = 3e^{2x}$

$\cancel{4Axe^{2x}} + 4Ae^{2x} \cancel{-6Axe^{2x}} - 3Ae^{2x} \cancel{+2Axe^{2x}} = 3e^{2x}$

$Ae^{2x} = 3e^{2x}$

$A=3$

$y_p = 3xe^{2x}$

Step 3 general solution is $y = y_c + y_p$ →

SOL 2: Variation of parameters

Step 1: Find y_c for $y''-3y'+2y=0$

← SAME as sol 1.

$y_c = c_1\underbrace{e^x}_{y_1} + c_2\underbrace{e^{2x}}_{y_2}$

Step 2: Find y_p (using variation of parameters)

(i) Standard Form ✓ $y''-3y'+2y = \underbrace{3e^{2x}}_{f(x)}$

$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^x & e^{2x} \\ e^x & 2e^{2x} \end{vmatrix} = (2e^{2x})(e^x) - (e^x)(e^{2x})$

$= 2e^{3x} - e^{3x} = e^{3x}$

$u_1 = \int \frac{-y_2 f(x)}{W} = \int -\frac{(e^{2x})(3e^{2x})}{e^{3x}}\,dx = -3\int e^x\,dx = -3e^x$

$u_2 = \int \frac{y_1 f(x)}{W} = \int \frac{e^x(3e^{2x})}{e^{3x}}\,dx = \int 3\,dx = 3x$

$y_p = u_1y_1 + u_2y_2 = (-3e^x)(e^x) + (3x)(e^{2x})$

$= -3e^{2x} + 3xe^{2x}$

Step 3: $y = y_c + y_p = c_1e^x + c_2e^{2x} + (-3e^{2x}) + 3xe^{2x}$

$= c_1e^x + d_2e^{2x} + 3xe^{2x}$

so ↙

General Solution: $\boxed{c_1e^x + c_2e^{2x} + 3xe^{2x}}$

2. Using the method of undetermined coefficients write the FORM for the particular solution (y_p) using the given value for $g(x)$ and the general solution of the associated homogeneous equation (y_c). Do NOT solve for the unknown constants, just write the form.

(a) $g(x) = 4e^x$ and $y_c = C_1e^x + C_2xe^x$. so

First guess: $y_p = Ae^x$ BUT e^x is part of $y_c \Rightarrow y_p = Axe^x$ BUT xe^x is part of $y_c \Rightarrow$ $y_p = Ax^2e^x$ ⟶

Form of y_p: $\boxed{Ax^2e^x}$

(b) $g(x) = 3\cos(2x)$ and $y_c = C_1\sin(2x) + C_2\cos(2x)$. so

First guess: $y_p = A\cos(2x) + B\sin(2x)$

But both $\cos(2x)$ and $\sin(2x)$ are part of y_c so

$y_p = Ax\cos(2x) + Bx\sin(2x)$ ⟶

Form of y_p: $\boxed{Ax\cos(2x) + Bx\sin(2x)}$

(c) $g(x) = 4e^x\sin 4x$ and $y_c = C_1e^x\sin(x) + C_2e^x\cos(x)$. so

First guess: $y_p = Ae^x\sin(4x) + Be^x\cos(4x)$

this is good! since neither $e^x\sin(4x)$ or $e^x\cos(4x)$ are part of y_c so ⟶

Form of y_p: $\boxed{Ae^x\sin(4x) + Be^x\cos(4x)}$