Math 271 - Fall 2012 - Quiz #5 **Score:** _____ out of 10.

Name: _____

Show all work clearly and in order. Please box your answers.

1. A cup of coffee is brewed at 195°F and sits on a desk at a room temperature of 73°F. 5 minutes later the temperature is 110°F. How long will it take for the coffee to cool off to a temperature of 80°F. Hint: you will use Newton's Law of Cooling/Warming:

$$\frac{dT}{dt} = k(T - T_m).$$

For this problem: $T_m = 73$

So solve the IVP: $\begin{cases} \frac{dT}{dt} = K(T-73) \\ T(0) = 195 \end{cases}$ and determine the value k so that $T(5) = 110$ and then determine the time t s.t. $T(t) = 80$.

$\frac{dT}{dt} = K(T-73)$ is

$\int \frac{dT}{T-73} = \int k$

$\ln|T-73| = Kt + C$

$|T-73| = e^{kt+C} = Ae^{kt}$

$T-73 = Be^{kt}$

$T = Be^{kt} + 73$

so $T(0) = 195 = Be^0 + 73$

$B = 122$

so $T(t) = 122e^{kt} + 73$

$T(5) = 110 = 122e^{k5} + 73$

$37 = 122e^{k5} \Rightarrow e^{k5} = 37/122$

$k5 = \ln(37/122)$ so $k \approx -0.2386$

so $T(t) = 122e^{-0.2386t} + 73$

SOLVE $T(t) = 80 = 122e^{-0.2386t} + 73 \Rightarrow 7 = 122e^{-0.2386t}$

$\frac{7}{122} = e^{-0.2386t}$

$\ln(7/122) = -0.2386t$

$t = 11.98$ minutes

Time (in minutes): **11.98 minutes**

2. **Career Fair!** A career fair is coming to Alfred University and word is getting around. Initially 10 students know about the career fair and start to tell the isolated campus of 2400 students. It is assumed that the rate at which the knowledge of the career fair spreads is proportional to not only the number x of students who know about the career fair, but also to the number of students who do not know about the career fair. Determine the number of students who know about the career fair after 5 days if it is further observed that after 3 days $x(3) = 250$. Hint: you will use the following logistic equation:

$$\begin{cases} \frac{dx}{dt} = kx(2400 - x). \\ x(0) = 10 \end{cases}$$

For this problem we need to solve the IVP: and determine k so that $x(3) = 250$ and then determine the number of students $x(5)$

$\frac{dx}{dt} = kx(2400-x)$

$\int \frac{dx}{x(2400-x)} = \int k\,dt$

$\frac{1}{x(2400-x)} = \frac{A}{x} + \frac{B}{2400-x}$

$1 = A(2400-x) + Bx$

$1 = 2400A - Ax + Bx$

$1 = 2400A + (-A+B)x$

so $A = \frac{1}{2400}$ and $-A+B = 0$

so $B = \frac{1}{2400}$

so $\frac{1}{2400}\int\left(\frac{1}{x} + \frac{1}{2400-x}\right)dx = kt + C$

$\frac{1}{2400}\left(\ln|x| - \ln|2400-x|\right) = kt + C$

$\ln\left|\frac{x}{2400-x}\right| = 2400kt + D$

$\left|\frac{x}{2400-x}\right| = e^{2400kt+D} = Ee^{2400kt}$

$\frac{x}{2400-x} = Fe^{2400kt}$

$x = (2400-x)Fe^{2400kt}$

$x = 2400Fe^{2400kt} - xFe^{2400kt}$

$x + xFe^{2400kt} = 2400Fe^{2400kt}$

$x(1 + Fe^{2400kt}) = 2400Fe^{2400kt}$

$x = \frac{2400Fe^{2400kt}}{1 + Fe^{2400kt}}$

CONTINUE!

Number of AU students nervous about the carrer fair after 5 days: **1240**

NOTE: rounding will affect your answer!!

$$X = \frac{2400Fe^{2400kt}}{e^{2400kt}\left[\frac{1}{e^{2400kt}} + F\right]} = \frac{2400F}{e^{-2400kt} + F}$$

$$x(0) = 10 = \frac{2400F}{F + e^{0}} = \frac{2400F}{F+1}$$

$$10F + 10 = 2400F$$

$$10 = 2390F$$

$$F = \frac{10}{2390} \approx 0.004184$$

$$x(t) = \frac{2400(0.004184)}{e^{-2400kt} + 0.004184} = \frac{10.0418}{e^{-2400kt} + 0.004184}$$

$$x(3) = 250 = \frac{10.0418}{e^{-2400k(3)} + 0.004184}$$

so $250\left(e^{-2400k(3)} + 0.004184\right) = 10.0418$

$$250e^{-7200k} + 1.046 = 10.0418$$

$$250e^{-7200k} = 8.9958$$

$$e^{-7200k} = 0.035983$$

$$-7200k = \ln(0.035983) = -3.3247$$

$$k = +0.000462$$

$$x(t) = \frac{10.0418}{e^{-2400(+0.000462)t} + 0.004184}$$

$x(5) = 1240.52$ so about **1240 people**