Math 271 - Fall 2012 - Quiz #2 **Score:** ______ out of 10.

Name: Key

Show all work clearly and in order. Please box your answers. 10 minutes.

1. Fill in the following table with the missing classification information:

ODE	order	linear/nonlinear
$y'' + 2xy = 1$	2	LINEAR
$\left(\frac{d^2y}{dx^2}\right)^4 + \left(\frac{dy}{dx}\right) = x$	2	NONLINEAR
$\sin(x)y^{(4)} + \ln(x)y' = \cos(x)$	4	LINEAR

2. (a) Verify that $y = \dfrac{1}{1+Ce^{-x}}$ is a one-parameter family of solutions to $y' = y - y^2$.

$\underbrace{y'}_{LHS} = \underbrace{y - y^2}_{RHS}$. Notice that $y = (1+Ce^{-x})^{-1}$

The Left Hand side (LHS) $= y' = (-1)(1+Ce^{-x})^{-2} \cdot (-Ce^{-x}) = \dfrac{Ce^{-x}}{(1+Ce^{-x})^2}$ (OR use quotient rule)

SAME!!

The Right Hand Side (RHS) $= y - y^2 = \left(\dfrac{1}{1+Ce^{-x}}\right) - \left(\dfrac{1}{1+Ce^{-x}}\right)^2 = \dfrac{1}{1+Ce^{-x}} - \dfrac{1}{(1+Ce^{-x})^2} = \dfrac{(1+Ce^{-x})-1}{(1+Ce^{-x})^2} = \dfrac{Ce^{-x}}{(1+Ce^{-x})^2}$

(b) Find a solution to the initial value problem (IVP) consisting of the differential equation $y' = y - y^2$ and the initial condidtion $y(-1) = 2$.

In part (a) we verified that $y = \dfrac{1}{1+Ce^{-x}}$ is the general solution of $y' = y - y^2$.

Hence, $y(-1) = \dfrac{1}{1+Ce^{-(-1)}} = \dfrac{1}{1+Ce^{1}} = \dfrac{1}{1+Ce} = 2$

$1 = 2 + 2Ce$

$-1 = 2Ce$

$C = \dfrac{-1}{2e}$. Therefore, the solution is:

$$\boxed{y = \dfrac{1}{1+\left(\frac{-1}{2e}\right)e^{-x}}}$$