[redacted]

$x=0$ is a regular singular point of

$$2xy'' + 5y' + xy = 0$$

use the method of Frobenius to find two linealy independent series solutions about $x=0$.

<u>Sol</u>:

we want solutions of the form

$$y = \sum_{n=0}^{\infty} c_n x^{n+r}$$

substituting

$$y' = \sum_{n=0}^{\infty} (n+r) c_n x^{n+r-1}$$

$$y'' = \sum_{n=0}^{\infty} (n+r)(n+r-1) c_n x^{n+r-2}$$

$$2xy'' + 5y' + xy = 0$$

$$2x \sum_{n=0}^{\infty} (n+r)(n+r-1)c_n x^{n+r-2} + 5 \sum_{n=0}^{\infty} (n+r) c_n x^{n+r-1} + x \sum_{n=0}^{\infty} c_n x^{n+r} = 0$$

$$\sum_{n=0}^{\infty} 2(n+r)(n+r-1) c_n x^{n+r-1} + \sum_{n=0}^{\infty} 5(n+r)c_n x^{n+r-1} + \sum_{n=0}^{\infty} c_n x^{n+r+1} = 0$$

starts x^{r-1} starts x^{r-1} starts x^{r+1}

$$2r(r-1)c_0 x^{r-1} + 2(1+r)r\, c_1 x^r + 5rc_0 x^{r-1} + 5(r+1)c_1 x^r +$$
$$\sum_{n=2}^{\infty} 2(n+r)(n+r-1)c_n x^{n+r-1} + \sum_{n=2}^{\infty} 5(n+r)c_n x^{n+r-1} + \sum_{n=0}^{\infty} c_n x^{n+r+1} = 0$$

$$(2r^2 - 2r + 5r)c_0 x^{r-1} + (2r + r^2 + 5r + 5)c_1 x^r +$$
$$\sum_{n=2}^{\infty} 2(n+r)(n+r-1)c_n x^{n+r-1} + \sum_{n=2}^{\infty} 5(n+r)c_n x^{n+r-1} + \sum_{n=2}^{\infty} c_{n-2} x^{n+r-1} = 0$$

minus

$$(2r^2+3r)c_0 x^{r-1} + (r^2+7r+5)c_1 x^r + \sum_{n=2}^{\infty}\left[2(n+r)(n+r-1)c_n + 5(n+r)c_n + c_{n-2}\right] x^{n+r-1} = 0$$

$(2r^2+3r)c_0 = 0$ <u>AND</u> $(r^2+7r+5)c_1 = 0$ <u>AND</u> $2(n+r)(n+r-1)c_n + 5(n+r)c_n + c_{n-2} = 0$ for $n=2,3,4,\ldots$

INDICIAL EQuation! (coef of the lowest power of x)

$$2r^2 + 3r = 0$$
$$r(2r+3) = 0$$
$$r=0 \text{ or } r = -3/2$$

For $r=0$

$c_1 = 0$

and the recurrence relation is

$$c_n = \frac{-c_{n-2}}{2(n+0)(n+0-1)+5(n+0)}$$

$$c_n = \frac{-c_{n-2}}{n(2n+3)}, \quad n=2,3,4,\ldots$$

$c_0 = ?$
$c_1 = 0$

n	$c_n = \frac{-c_{n-2}}{n(2n+3)}$
2	$-\frac{1}{14}c_0$
3	0
4	$\frac{1}{616}c_0$

solution is of the form

$$y = \sum_{n=0}^{\infty} c_n x^{n+r}$$

$$= x^r \sum_{n=0}^{\infty} c_n x^n$$

for $r=0$

$$y = x^0\left(\sum_{n=0}^{\infty} c_n x^n\right)$$

$$= x^0\left(c_0 + c_1x + c_2x^2 + c_3x^3 + \cdots\right)$$

$$= 1\left(c_0 + 0x + -\frac{1}{14}c_0x^2 + 0 + \frac{1}{616}c_0x^4 + \cdots\right)$$

$$= c_0 - \frac{1}{14}c_0x^2 + \frac{1}{616}c_0x^4 + \cdots$$

$$= c_0\left(1 - \frac{1}{14}x^2 + \frac{1}{616}x^4 + \cdots\right)$$

first solution

For $r = -3/2$

$c_1 = 0$

recurrence relation is

$$c_n = -\frac{c_{n-2}}{n(2n-3)}, \quad n=2,3,4,\ldots$$

$c_0 = ?$
$c_1 = 0$

n	$c_n = \frac{-c_{n-2}}{n(2n-3)}$
2	$= \frac{-c_0}{2(1)} = \frac{-c_0}{2}$
3	0
4	$\frac{-c_2}{4\cdot 5} = \frac{c_0}{4\cdot 5\cdot 2} = \frac{c_0}{40}$

for $r = -3/2$

$$y = x^{-3/2}\sum_{n=0}^{\infty} c_n x^n$$

$$= x^{-3/2}\left(c_0 + c_1x + c_2x^2 + \cdots\right)$$

$$= x^{-3/2}\left(c_0 + 0 + -\frac{1}{2}c_0x^2 + 0 + \frac{c_0}{40}x^4 \cdots\right)$$

$$= c_0 x^{-3/2}\left(1 - \frac{1}{2}x^2 + \frac{1}{40}x^4 + \cdots\right)$$

second solution.