Math 271 - Quiz #4 **Score:** ______ out of 10.

Name: ______________________________

Show all work clearly and in order. Please box your answers. 10 minutes.

Choose ONE side. Clearly put an X on the side you do not want me to grade, otherwise I will grade the first side worked on.

1. Find two power series solutions of the given differential equation centered about the ordinary point $x = 0$.

$$y'' - 2xy' + y = 0$$

$$y = \sum_{n=0}^{\infty} c_n x^n$$

$$y' = \sum_{n=1}^{\infty} n c_n x^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} n(n-1) c_n x^{n-2}$$

$$y'' - 2xy' + y = 0$$

bring in coef.

$$\sum_{n=2}^{\infty} n(n-1) c_n x^{n-2} - 2x \sum_{n=1}^{\infty} n c_n x^{n-1} + \sum_{n=0}^{\infty} c_n x^n = 0$$

$$\sum_{n=2}^{\infty} n(n-1) c_n x^{n-2} - \sum_{n=1}^{\infty} 2n c_n x^n + \sum_{n=0}^{\infty} c_n x^n = 0$$

Starts x^0 Starts x^1 starts x^0

$$2(1)c_2 x^0 + \sum_{n=3}^{\infty} n(n-1)c_n x^{n-2} - \sum_{n=1}^{\infty} 2n c_n x^n + c_0 x^0 + \sum_{n=1}^{\infty} c_n x^n = 0$$

shift to n = 1

$$(c_0 + 2c_2) + \sum_{n=1}^{\infty} (n+2)(n+1) c_{n+2} x^n - \sum_{n=1}^{\infty} 2n c_n x^n + \sum_{n=1}^{\infty} c_n x^n = 0$$

$$(c_0 + 2c_2) + \sum_{n=1}^{\infty} \left[(n+2)(n+1) c_{n+2} - 2n c_n + c_n \right] x^n = 0$$

$c_0 + 2c_2 = 0$ **AND** $(n+2)(n+1)c_{n+2} - 2n c_n + c_n = 0$ for $n = 1, 2, 3, \dots$

$c_2 = \frac{-c_0}{2}$ $\qquad c_{n+2} = \frac{2n c_n - c_n}{(n+2)(n+1)} = \frac{(2n-1)}{(n+2)(n+1)} c_n$ for $n = 1, 2, 3, \dots$

TABLE

$y_1 = \boxed{1 - \frac{x^2}{2!} - \frac{3}{4!}x^4 - \frac{21}{6!}x^6 + \cdots}$

$y_2 = \boxed{x + \frac{1}{3!}x^3 + \frac{5}{5!}x^5 + \cdots}$

$C_0 = ?$

$C_1 = ?$

$C_2 = -\frac{C_0}{2}$

n	$C_{n+2} = \frac{(2n-1)}{(n+2)(n+1)} C_n$
1	$C_3 = \frac{1}{3\cdot 2} C_1 = \frac{1}{3!} C_1$
2	$C_4 = \frac{3}{4\cdot 3} C_2 = \frac{3}{4\cdot 3}\left(-\frac{C_0}{2}\right) = \frac{-3}{4\cdot 3\cdot 2} C_0 = \frac{-3}{4!} C_0$
3	$C_5 = \frac{5}{5\cdot 4} C_3 = \frac{5}{5\cdot 4}\left(\frac{1}{3\cdot 2}\right) C_1 = \frac{5}{5\cdot 4\cdot 3\cdot 2} C_1 = \frac{5}{5!} C_1$
4	$C_6 = \frac{7}{6\cdot 5} C_4 = \frac{7}{6\cdot 5}\left(\frac{-3}{4\cdot 3\cdot 2} C_0\right) = \frac{-7\cdot 3}{6\cdot 5\cdot 4\cdot 3\cdot 2} C_0 = \frac{-7\cdot 3}{6!} C_0$
$\vdots$	$\vdots$

$$y = \sum_{n=0}^{\infty} C_n x^n = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + C_4 x^4 + \cdots$$

$$= C_0 + C_1 x + \left(-\frac{C_0}{2}\right)x^2 + \left(\frac{1}{3!}C_1\right)x^3 + \left(\frac{-3}{4!}C_0\right)x^4 + \left(\frac{5}{5!}C_1\right)x^5 + \left(\frac{-7\cdot 3}{6!}C_0\right)x^6 + \cdots$$

$$= \left(C_0 - \frac{C_0}{2}x^2 + \left(\frac{-3}{4!}C_0\right)x^4 + \left(\frac{-7\cdot 3}{6!}C_0\right)x^6 + \cdots\right) +$$

$$\left(C_1 x + \left(\frac{1}{3!}C_1\right)x^3 + \left(\frac{5}{5!}C_1\right)x^5 + \cdots\right)$$

$$= C_0\Big(\underbrace{1 - \frac{x^2}{2} - \frac{3}{4!}x^4 - \frac{7\cdot 3}{6!}x^6 + \cdots}_{y_1}\Big) + C_1\Big(\underbrace{x + \frac{1}{3!}x^3 + \frac{5}{5!}x^5 + \cdots}_{y_2}\Big)$$

note: you may include this in y_1

note: you may include this in y_2

2. Use the Laplace transform to solve the following initial value problem:

$$y' + 6y = e^{4t}, \quad y(0) = 2$$

$$\mathcal{L}\{y' + 6y\} = \mathcal{L}\{e^{4t}\}$$

$$\mathcal{L}\{y'\} + 6\mathcal{L}\{y\} = \mathcal{L}\{e^{4t}\}$$

$$(sY(s) - y(0)) + 6Y(s) = \frac{1}{s-4}$$

$$sY(s) - 2 + 6Y(s) = \frac{1}{s-4}$$

$$sY(s) + 6Y(s) = \frac{1}{s-4} + 2 = \frac{1 + 2(s-4)}{s-4} = \frac{1 + 2s - 8}{s-4}$$

$$Y(s)(s+6) = \frac{-7+2s}{s-4}$$

$$Y(s) = \frac{-7+2s}{(s-4)(s+6)} = \frac{A}{s-4} + \frac{B}{s+6} = \frac{A(s+6) + B(s-4)}{(s-4)(s+6)}$$

$$-7 + 2s = As + 6A + Bs - 4B$$

$$-7 + 2s = (A+B)s + 6A - 4B$$

$$A + B = 2 \qquad -7 = 6A - 4B$$

$$A = 2 - B \rightarrow -7 = 6(2-B) - 4B$$

$$-7 = 12 - 6B - 4B$$

$$-19 = -10B$$

$$A = 2 - \frac{19}{10} \leftarrow \underline{B = \frac{19}{10}}$$

$$\underline{A = \frac{1}{10}}$$

$$Y(s) = \frac{1/10}{s-4} + \frac{19/10}{s+6}$$

$$y = \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{1/10}{s-4} + \frac{19/10}{s+6}\right\} = \frac{1}{10}e^{4t} \quad \frac{19}{10}e^{-6t}$$

$y(t) = \boxed{\frac{1}{10}e^{4t} + \frac{19}{10}e^{-6t}}$