Math 271 - Quiz #2 **Score:** _______ out of 10.

Name: ~~Key~~

Show all work clearly and in order. Please box your answers. 10 minutes.

Choose ONE side. Clearly put an X on the side you do not want me to grade, otherwise I will grade the first side worked on.

1. The function $y_1 = x^4$ is a solution to $x^2y'' - 7xy' + 16y = 0$. Use the reduction of order equation formula to find a second solution $y_2(x)$. (NOTE: you do not need to vertify that y_1 is a solution, just find y_2.)

Standard Form: $y'' \underbrace{- \frac{7}{x}}_{P(x)} y' + \frac{16}{x^2} y = 0$, assume $x > 0$

$$y_2 = y_1 \int \frac{e^{-\int P(x)dx}}{(y_1)^2} dx = x^4 \int \frac{e^{-\int(-7/x)dx}}{(x^4)^2} dx$$

$$= x^4 \int \frac{e^{7\ln(x)}}{x^8} dx$$

$$= x^4 \int \frac{e^{\ln(x^7)}}{x^8} dx$$

$$= x^4 \int \frac{x^7}{x^8} dx = x^4 \int \frac{1}{x} dx = \boxed{\ln(x) x^4}$$

2. Determine whether the given set of functions is linearly independent on the interval $(0, \infty)$. SHOW WORK AND CLEARLY STATE whether the set of functions is **linearly independent** or **linearly dependent.**

(a) $f_1(x) = x$, $f_2(x) = x\ln(x)$

$$W = \begin{vmatrix} f_1 & f_2 \\ f_1' & f_2' \end{vmatrix} = \begin{vmatrix} x & x\ln(x) \\ 1 & x(\frac{1}{x}) + \ln(x) \end{vmatrix} = x(1+\ln(x)) - x\ln(x)$$
$$= x \neq 0 \text{ on } (0,\infty)$$

Hence, f_1 and f_2 are $\boxed{\text{linearly independent.}}$

(b) $g_1(x) = 2$, $g_2(x) = \sec^2(x)$, $g_3(x) = 1 - 12\sec^2(x)$

Sol 1: $\frac{1}{2}\underbrace{(2)}_{g_1} - 12\underbrace{(\sec^2(x))}_{g_2} - 1\underbrace{(1 - 12\sec^2(x))}_{g_3} = 1 - 12\sec^2(x) - 1 + 12\sec^2(x) = 0$

Hence, since the coefficients $\frac{1}{2}, -12, -1$ are NOT ALL 0, g_1, g_2 and g_3 are $\boxed{\text{linearly dependent}}$

Sol: Show the Wranskian is 0 on $(0,\infty)$, Hence

3. Find the general solution to the following:

(a) $y'' + 5y' + 6y = 0$

$$m^2 + 5m + 6 = 0$$
$$(m+3)(m+2) = 0$$
$$m = -3 \quad | \quad m = -2$$

$$\boxed{y = C_1 e^{-3x} + C_2 e^{-2x}}$$

(b) $y^{(4)} - 6y''' + 9y'' = 0$

$$m^4 - 6m^3 + 9m^2 = 0$$
$$m^2(m^2 - 6m + 9) = 0$$
$$m^2(m-3)(m-3) = 0$$
$$m=0 \quad | \quad m=0 \quad | \quad m=3 \quad | \quad m=3$$

$$y = C_1 e^{0x} + C_2 x e^{0x} + C_3 e^{3x} + C_4 x e^{3x}$$
$$\boxed{y = C_1 + C_2 x + C_3 e^{3x} + C_4 x e^{3x}}$$

(c) $y^{(4)} - 81y = 0$

$$m^4 - 81 = 0$$
$$(m^2 - 9)(m^2 + 9) = 0$$
$$(m-3)(m+3)(m^2+9) = 0$$
$$m = 3 \quad | \quad m = -3 \quad | \quad \begin{array}{l} m^2 = -9 \\ m = \pm\sqrt{-9} = \pm 3i \\ \alpha = 0, \quad \beta = 3 \end{array}$$

$$y = C_1 e^{3x} + C_2 e^{-3x} + C_3 e^{0x}\cos(3x) + C_4 e^{0x}\sin(3x)$$
$$\boxed{y = C_1 e^{3x} + C_2 e^{-3x} + C_3 \cos(3x) + C_4 \sin(3x)}$$