

Name: Key

Show all work clearly and in order. Please box your answers. 10 minutes.

1. Fill in the following table with the missing classification information:

DE	order	linear/nonlinear
$xy'' - y' + 2y = x$	2	linear
$\tan(x)y^{(10)} + \ln(x)y'' = \cot(x)$	10	linear
$\frac{d^5y}{dx^5} + xy^4 = \sin(x)$	5	nonlinear

2. Verify that $y = \sin(3x)$ is a solution to the differential equation $y'' + 9y = 0$.

$$\begin{aligned} y &= \sin(3x) \\ y' &= 3\cos(3x) \\ y'' &= -9\sin(3x) \end{aligned} \quad \left. \begin{array}{l} \rightarrow \\ \text{---} \end{array} \right. \begin{aligned} y'' + 9y &= 0 \\ -9\sin(3x) + 9\sin(3x) &= 0 \\ 0 &= 0 \quad \checkmark \end{aligned}$$

3. Find the value(s) of m so that $y = x^m$ is a solution to the differential equation $xy'' + 2y' = 0$.

$$\begin{aligned} y' &= mx^{m-1} \\ y'' &= m(m-1)x^{m-2} \end{aligned}$$

$$\begin{aligned} xy'' + 2y' &= 0 \\ x m(m-1)x^{m-2} + 2mx^{m-1} &= 0 \\ m(m-1)x^{m-1} + 2mx^{m-1} &= 0 \\ (m^2 - m + 2m)x^{m-1} &= 0 \\ (m^2 + m)x^{m-1} &= 0 \quad \text{if } x \neq 0 \text{ then} \end{aligned}$$

$$\begin{aligned} m^2 + m &= 0 \\ m(m+1) &= 0 \end{aligned} \quad \rightarrow \quad \boxed{m=0 \quad \underline{\text{or}} \quad m=-1}$$