TEST 2

Math 271 - Differential Equations **Score:** _________ out of 100

3/19/2014 **Name:** ______________________________

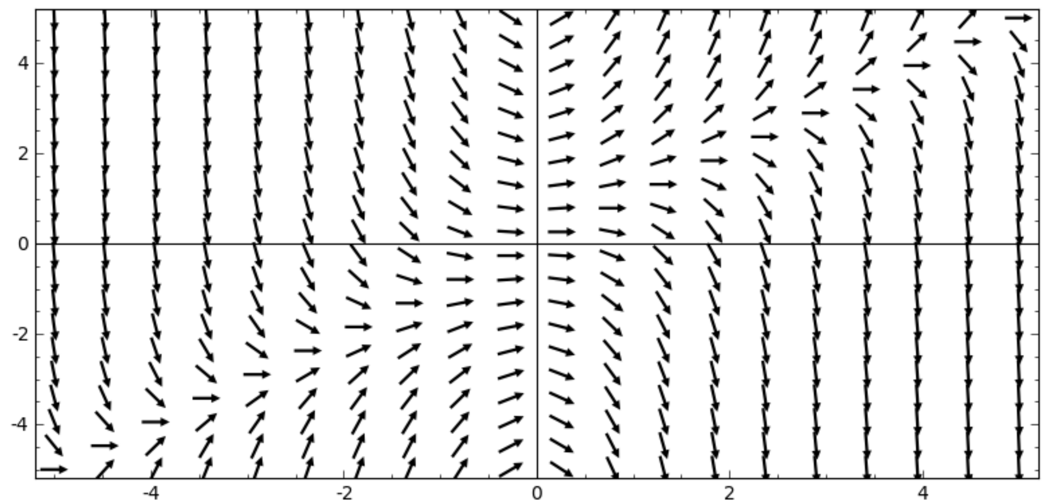
Read all of the following information before starting the exam:

- You have 50 minutes to complete the exam.
- Show all work, clearly and in order, if you want to get full credit. Please make sure you read the directions for each problem. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Please box/circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 7 problems and is worth 100 points. It is your responsibility to make sure that you have all of the pages!
- Good luck!

1. The following is the direction field for the differential equation

$$\frac{dy}{dx} = xy - x^2,$$

over the region $R = \{(x, y) \mid -5 \le x \le 5, -5 \le y \le 5\}$.



Sketch an approximate solution curve that passes through the following points:

(a) $y(0) = 0$.

(b) $y(0) = 1$

Use your solution curve that passes through the point $y(0) = 0$ to estimate the value of $y(-2)$.

$$y(-2) =$$

2. The function $y_1 = \ln(x)$ is a solution to $xy'' + y' = 0$. Use the reduction of order equation formula to find a second solution $y_2(x)$. (NOTE: you do not need to vertify that y_1 is a solution, just find y_2.)

3. Determine whether the given set of functions is linearly independent on the interval $(0, \infty)$. SHOW WORK AND CLEARLY STATE whether the set of functions is **linearly independent** or **linearly dependent**.

   (a) $f_1(x) = e^{2x}$, $f_2(x) = e^{3x}$

   (b) $g_1(x) = -\sin^2(x)$, $g_2(x) = 2\cos^2(x)$, $g_3(x) = 3$

4. Complete all of the following parts. **You may not use the auxiliary/characteristic equation method!**

   (a) Verify that $y_1 = x$ and $y_2 = x\ln(x)$ form a fundamental set of solutions of $x^2y'' - xy' + y = 0$ on $(0, \infty)$.

   (b) Verify that $y_p = 2 + \ln(x)$ forms a particular solution of $x^2y'' - xy' + y = \ln(x)$.

   (c) Use (a) and (b) to write the general solution of $x^2y'' - xy' + y = \ln(x)$.

General Solution: | |
|---|

5. Find the general solution to the following:

(a) $y'' - 4y' + 5y = 0$

(b) $y''' + 2y'' - 4y' - 8y = 0$

(c) $y^{(6)} - 9y^{(4)} = 0$

6. Solve the following differential equation using the method of undetermined coefficients:

$$y'' + 3y' + 2y = 4x^2$$

General Solution:

7. Solve the following differential equation using the variation of parameters:

$$y'' - 4y' + 4y = (x + 1)e^{2x}$$

Hint: make sure you simplify the Wronskian!

General Solution: