PRACTICE TEST 2

Math 271 - Differential Equations **Score:** _________ out of 100

Name: ______________________________

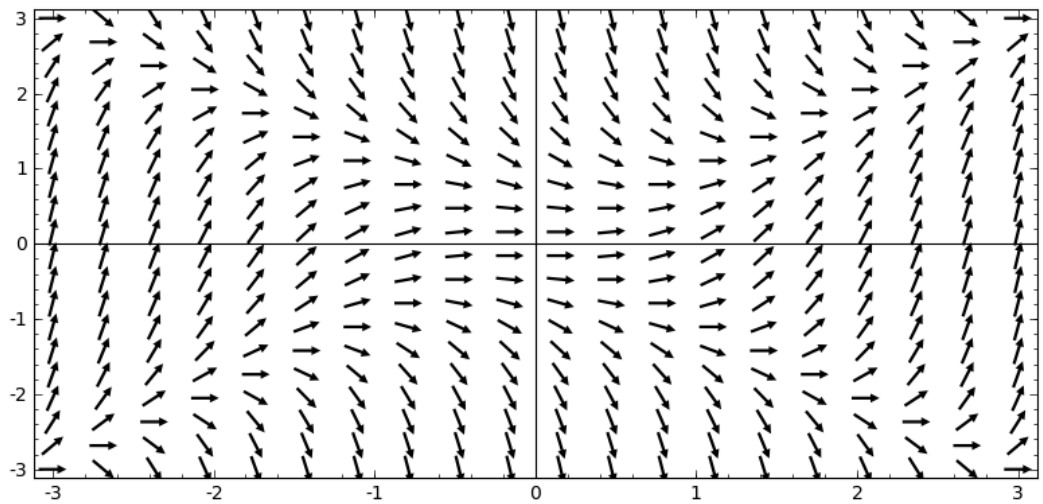
Read all of the following information before starting the exam:

- You have 50 minutes to complete the exam.
- Show all work, clearly and in order, if you want to get full credit. Please make sure you read the directions for each problem. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Please box/circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 7 problems and is worth 100 points. It is your responsibility to make sure that you have all of the pages!
- Good luck!

1. The following is the direction field for the differential equation

$$\frac{dy}{dx} = x^2 - y^2,$$

over the region $R = \{(x, y) \mid -3 \le x \le 3, -3 \le y \le 3\}$.



Sketch an approximate solution curve that passes through the following points:

(a) $y(-2) = 1$.

(b) $y(3) = 0$

Use your solution curve that passes through the point $y(-2) = 1$ to estimate the value of $y(2)$.

$$y(2) =$$

2. The function $y_1 = x^4$ is a solution to $x^2y'' - 7xy' + 16y = 0$. Use the reduction of order equation formula to find a second solution $y_2(x)$. (NOTE: you do not need to vertify that y_1 is a solution, just find y_2.)

3. Determine whether the given set of functions is linearly independent on the interval $(0, \infty)$. SHOW WORK AND CLEARLY STATE whether the set of functions is **linearly independent** or **linearly dependent**.

   (a) $f_1(x) = x,\ f_2(x) = x \ln(x)$

   (b) $g_1(x) = 5,\ g_2(x) = \sin(x),\ g_3(x) = 10 - 7\sin(x)$

4. Complete all of the following parts. **You may not use the auxiliary/characteristic equation method!**

   (a) Verify that $y_1 = e^{-x}$ and $y_2 = e^{x}$ form a fundamental set of solutions of $y'' - y = 0$ on $(-\infty, \infty)$.

   (b) Verify that $y_p = \dfrac{1}{8}e^{3x}$ forms a particular solution of $y'' - y = e^{3x}$.

   (c) Use (a) and (b) to write the general solution of $y'' - y = e^{3x}$.

General Solution: | |
|---|

5. Find the general solution to the following:

(a) $y'' + y' - 12y = 0$

(b) $y''' - 4y'' + 4y' = 0$

(c) $y^{(4)} - 16y = 0$

6. Solve the following differential equation using the <u>method of undetermined coefficients</u>:

$$y'' + y' - 2y = 5e^x$$

General Solution: | |

7. Solve the following differential equation using the variation of parameters:

$$y'' + y = \sec(x)\tan(x)$$

General Solution: