

Slope Approximations

Slope Approximations is a graphics oriented program that numerically calculates secant line slopes. When the program is first executed the example shown in Figure 1 is already set up.

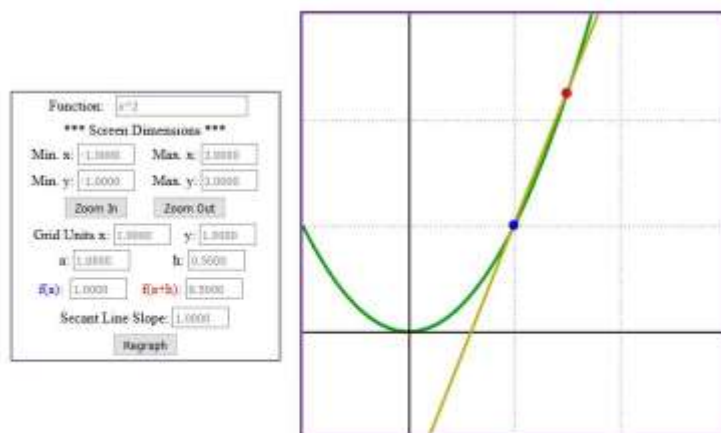


Figure 1

The graph depicts the curve $y = f(x) = x^2$ (the green curve) and its secant line (the yellow line) passing through the points $(a, f(a))$ and $(a + h, f(a + h))$, where $a = 1$ and $h = 0.5$. The slope of the secant line is calculated as $\frac{f(a+h)-f(a)}{h}$ and in this case the slope is 2.5. As h approaches zero, the secant lines look more and more like the tangent line at $(a, f(a))$, and corresponding secant line slopes approach the slope of the tangent line. In other words, the slope of the tangent line at $(a, f(a))$ is $\lim_{h \rightarrow 0} \frac{f(a+h)-f(a)}{h}$. In many cases this limit can be evaluated algebraically, but often the limit can also be deduced by creating a table. For example, consider the following:

h	$\frac{f(1+h)-f(1)}{h}$
0.1	2.1000
0.01	2.0100
0.001	2.0010
0.0001	2.0001
-0.1	1.9000
-0.01	1.9900
-0.001	1.9990
-0.0001	1.9999

This table suggests $\lim_{h \rightarrow 0} \frac{f(1+h)-f(1)}{h} = 2$. The program verifies this suggestion by setting $h = 0$. The corresponding tangent line graph is shown in Figure 2.

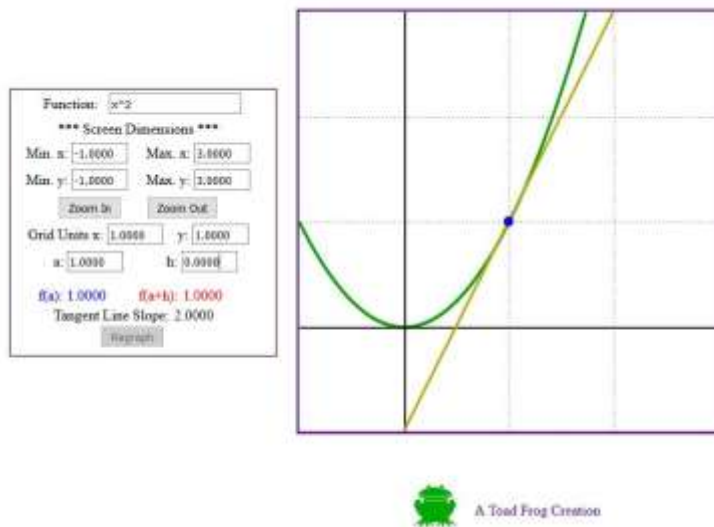
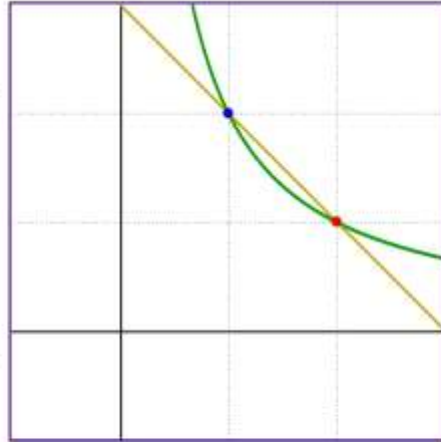
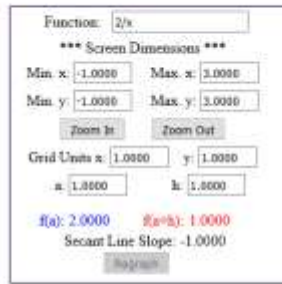


Figure 2

Here are some notes about the operation of this program:

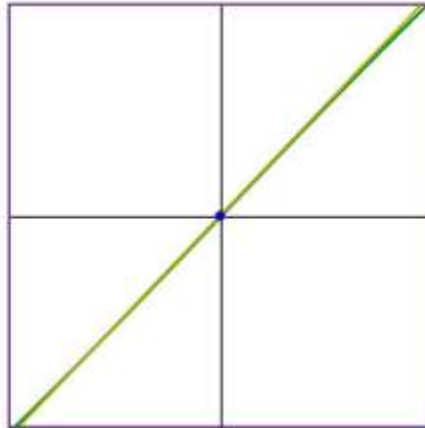
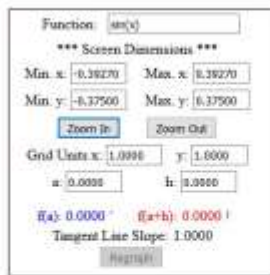
- When any of the text boxes above the “Zoom In” and “Zoom Out” buttons are edited, the graph is not automatically redrawn. Instead, the “Zoom In” and “Zoom Out” buttons are disabled while the “Regraph” button at the bottom of the screen is enabled. You can continue to edit any of the other text boxes and the graph will be redrawn once the “Regraph” button is pressed. This feature is useful when a completely new example is being set up.
- When the “Zoom In” button is pressed, the whole graph is re-centered around the point $(a, f(a))$. The graph is not re-centered when the “Zoom Out” button is pressed.
- An alternative way to change the value of h is by dragging the mouse. (This option does work on mobile devices.) Only the horizontal position of the mouse is used to determine the value of h .

Now let’s consider a few more examples. First, let $y = f(x) = 2/x$. Figure 3 shows the result when graphed over the $[-1, 3] \times [-1, 3]$ viewing rectangle and when $a = 1$ and $h = 1$. Next, let $y = f(x) = \sin(x)$, initially over the viewing rectangle $[-\pi/2, \pi/2] \times [-3/2, 3/2]$ and with $a = 0$. After the “Zoom In” button is pressed once and h is set to 0.0, the image in Figure 4 is displayed. Note that the tangent line slope is one and in this viewing rectangle, the graph of the tangent line is almost indistinguishable from the graph of $y = f(x) = \sin(x)$. This example is one justification for the assertion that for relatively small x , it is the case that $\sin(x) \approx x$. Finally consider $y = f(x) = e^x$. Figure 5 shows the display over the viewing rectangle $[-1, 3] \times [-1, 9]$ and where $a = 1$ and $h = 0$. Note that the value of the tangent line slope at $x = a = 1$ is exactly the same as $f(1)$. This is no accident, indeed for the function $f(x) = e^x$ it is the case that the slope of the tangent line for any value of $x = a$ is the same as $f(a) = e^a$.



 A Toad Frog Creation

Figure 3



 A Toad Frog Creation

Figure 4

Function:

*** Screen Dimensions ***

Min. x: Max. x:

Min. y: Max. y:

Grid Units x: y:

x: h:

f(x): 2.7183 f(x+h): 2.7183
Tangent Line Slope: 2.7183

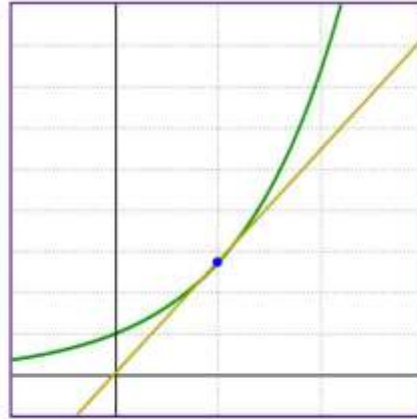


Figure 5