Iterate

Iterate is a program that explores the dynamics of iterating a simple function $f(x)$. It operates in five modes:

1. The Cob Web Mode, where the iterates $x_{n+1} = f(x_n)$ are analyzed through a cob web diagram.
2. Time Series Mode, which plots a time series graph, that is, the points $(n, f(x_n))$.
3. Histogram Mode, which plots a histogram, or bar chart, depicting the frequency which the $f(x_n)$ values fall into certain intervals.
4. Split Screen Mode, which plots one of the following side-by-side options:
   A. a cob web diagram along with a corresponding time series graph,
   B. a cob web diagram along with a corresponding histogram, or
   C. a time series graph along with a corresponding histogram.
5. Bifurcation Mode which creates bifurcation diagrams for various types of functions.

Upon execution of Iterate, the user is first asked to choose a mode. After that a user can change modes by selecting the appropriate option under the Mode menu.

Let’s begin by choosing the Cob Web mode. Upon selecting that mode, two windows will appear, as in Figure 1. The blank one on the left is the graph window and is where the cob web diagram will be displayed. The window on the right is the status window.
The program is initially set up to produce a cob web diagram for the Tent Map $T(x) = \frac{s}{2} - s \left| x - \frac{1}{2} \right|$, where $s$ is a parameter initially set to the value of 1.5. You can choose two other pre-defined maps:

1. The Logistic map: $L(x) = s \cdot x \cdot (1 - x)$
2. The Sine map: $S(x) = \frac{s}{\pi} \cdot \sin(\pi \cdot x)$

or you have the option of defining your own map. The seed is the initial $x$-value, $x_0$, and the depth is the order of the function being applied. For example, with depth = 2, each point plotted is the result of two additional iterations of the function. So with depth = 2 and the Tent map as rule, it is the function $T(T(x))$ that is examined. Upon pressing the “Go!” button your screen should look like Figure 2.

The blue line is the graph of the line $y = x$, the red curve is the graph of the Tent map, and the dotted lines are the top and left boundaries of the unit square $[0, 1] \times [0,1]$. Figure 3 shows the generation of the cob web diagram after the “Next” button was pressed ten times.

As an example of the Homemade Map, suppose you invest $100 at an annual interest rate of 8\% and you wish to know how long it will take for your investment to double. Select the “Homemade Map” option under the Rule menu. The function you need is $f(x) = (1 + s)x$, where the parameter $s$ is the interest rate. For this example $s = 0.08$. After you have defined the map and set the parameter. Your screen should look like Figure 4. Select “OK”, then in the status window change the seed value to 100 and the screen $x$- and $y$-intervals to $[-10, 220]$. Note that under normal circumstances the screen $x$- and $y$-intervals should be the same. Then press the “Go!” button. After pressing the
“Next” button nine times, you should see at the bottom of the screen that the $x$ value is 199.90046. Thus after nine years, your investment will grow to $199.90. If you press the “Next” button one more time, you will see that the investment will grow to $215.89.

Figure 3

Figure 4
Now let’s select the Time Series option on the Mode menu. The screen should look like Figure 5. Compared to the status window for the Cob Web mode, there are two additional items. The first is the “Spacing” text box. When spacing is $k$, then the vertical spacing between two successive time series points $(n, f(n))$ and $(n + 1, f(n + 1))$ is $k$ pixels. The default spacing is 10. The second item is a “Line Drawing Mode” selection box. There are three options:

1. Dot mode, where a small dot is plotted at each time series point $(n, f(n))$.
2. Single mode, where two successive time series points are connected by a straight line, and
3. Knot mode, which combines the Dot and Single modes. That is, a small dot is plotted at each time series point and two successive time series points are connected by straight lines.

Iterate automatically determines how many time series points it can plot without running over the right side of the graph window. When that number of points has been plotted, you are given three choices: 1) readjust the left and right boundaries of the graph window and plot more points, 2) trace the points currently showing, and 3) stop. Figure 6 shows the Time Series graph for the Logistic map with parameter 3.75 and graphed in the Dot mode.
Upon entering Histogram mode your screen should look like Figure 7. Compared to the initial status window for the Time Series mode, there are a couple of differences. First, instead of a “Spacing” text box, there is a “Number of Bars” text box. The screen y-interval is divided into that many subintervals of equal length to create the histogram classes. The second difference is that there is no screen x-interval. Figure 8 shows the resulting histogram when the tent map with parameter 1.5 is run with 1000 iterations and 120 histogram bars covering the screen y-interval $[-0.1, 1.1]$. If you click in the graph window, an arrow will appear just to the left of the vertical line. You can use the mouse to move the arrow up or down. At the bottom of the screen is a window which gives the minimum and maximum y values for the histogram class at which the arrow is pointing. It also gives the frequency of occurrences for that class. For the example shown the smallest y-value computed is no less than 0.37 and the largest is no greater than 0.76. The frequency class with the most occurrences of computed y-values (43) is the class corresponding to the interval $[0.62, 0.63]$.

Upon entering the Split Screen mode, you will be given three options:
1. Cob Web vs. Time Series
2. Cob Web vs. Histogram
3. Time Series vs. Histogram

Note that the “Save Graph” option in the Action menu is inoperative when in the Split Screen mode.

If you choose the first of the above options, then after pressing the “Go” button, your screen should look like Figure 9. The graph on the left is the start of a Cob Web diagram, whereas the window in the middle is where a Time Series graph will be plotted. Each time the “Next” button is pressed, another iteration will be generated.
Figure 10 shows the generation of each of these plots after “Next” button is pressed ten times. (The Line Drawing Mode was switched to “Knot”, just for variety.) Note that the screen $y$-interval is the same on both diagrams. The “Zoom” option only applies to the Cob Web diagram, but when the “Zoom” option is used and the screen $y$-interval is changed as a result, the Time Series screen $y$-interval will be automatically adjusted. The “Trace” option only applies to the Time Series diagram.
The second split screen display option graphs a cob web graph in the far left window with a corresponding histogram in the middle window. If you select the logistic function with
parameter value 3.1 and 24 histogram bars, then your screen should look like Figure 11 after the “Go” button is pressed and then the “Next” button is pressed 50 times. As in the previous split screen mode, the “Zoom” option only applies to the Cob Web diagram, but when the “Zoom” option is used and the screen y-interval is changed as a result, the Histogram screen y-interval will be automatically adjusted. Also you can click the mouse in the Histogram window in order to find out the frequency of occurrences in each of the histogram classes.

The third split screen option plots a time series graph in the far left window and a corresponding histogram in the middle window. If you choose that option and use the same options as before, (logistic map, parameter values equal to 3.1, number of iterations equal to 50, number of bars equal to 24), your screen should look like Figure 12 after pressing the “Go” button. The “Trace” option only applies to the Time Series graph and as before you can click the mouse in the Histogram window in order to find out the frequency of occurrences in each of the histogram classes.

The last mode to consider is the Bifurcation mode. In this mode the horizontal axis represents the parameter $s$. For each $s$ value, the iterates $x_{n+1} = f(x_n)$ are graphed along a vertical line. Figure 13 shows the bifurcation diagram for the Logistic map. Here the parametric interval is $[0,4]$, which means that the left side of the graph window corresponds to the parameter $s = 0$ and the right side of the graph corresponds to the parameter $s = 4$. The dynamical interval is $[-1,1.1]$ which means that the bottom of the graph window correspond to $x$ value -.1 and the top of the screen correspond to the $x$ value of 1.1. The seed is the initial iterate $x_0 = .5$. Iterates = 300 mean that for each value of the parameter $s$, 300 iterates $x_{n+1} = f(x_n)$ will be calculated. Depth = $k$ means
that the function $f^k(x)$ is being iterated. The default value of $k$ is 1. Drop = 150 means that for each value of the parameter $s$, the values of the first 150 iterations will not be plotted. Thus in this example, only the iterations $x_n$, $n = 151, 152, \ldots, 300$ will be plotted.

Figure 12

Figure 13
If you click the mouse in the graph window after the bifurcation diagram is plotted, a cursor will appear and the coordinates of the cursor will be displayed below the graph window. As you move the cursor around with the mouse, the coordinate display will be updated. Another mouse click will erase the cursor and the coordinate display.

Figure 14 shows the bifurcation diagram for the same mapping, parameter interval was changed to [3.4, 3.6] and the dynamical interval was changed to [32, 91].

![Figure 14](image_url)