Activity 6.4 – Inverse Trigonometric Functions

1. (a) $\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$

   (b) $\frac{d}{dx}\left(\cos^{-1} x\right) = -\frac{1}{\sqrt{1-x^2}}$

   (c) $\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$

2. (a) $\frac{d}{dx}\left(\sin^{-1}\left(e^x\right)\right) = \frac{1}{\sqrt{1-\left(e^x\right)^2}} \cdot e^x = \frac{e^x}{\sqrt{1-\left(e^x\right)^2}}$

   (b) $\frac{d}{dx}\left(x^2 \arccos\left(x^5\right)\right) = (2x)\left(\arccos\left(x^5\right)\right) + \left(x^2\right)\left(-\frac{1}{\sqrt{1-\left(x^5\right)^2}} \cdot 5x^4\right) = 2x \arccos\left(x^5\right) - \frac{5x^6}{\sqrt{1-x^{10}}}$

   (c) $\frac{d}{dx}\left(\frac{\tan^{-1}(x)}{\ln x}\right) = \frac{\left(\frac{1}{1+x^2}\right)(\ln x) - \left(\tan^{-1}(x)\right)\left(\frac{1}{x}\right)}{(\ln x)^2}$

3. (a) $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$

   (b) $\int \frac{-1}{\sqrt{1-x^2}} dx = \arccos x + C$

   (c) $\int \frac{1}{1+x^2}\, dx = \arctan x + C$

4. (a) $\int \frac{-1}{\sqrt{1-(3x)^2}}\, dx = \frac{1}{3}\arccos(3x) + C$

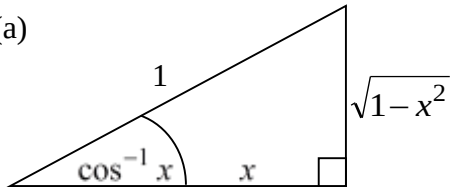
   (b) $\int_1^2 \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}} dx = 2\arcsin\left(\frac{x}{2}\right)\Big|_1^2 = 2\arcsin(1) - 2\arcsin\left(\frac{1}{2}\right) = 2 \cdot \frac{\pi}{2} - 2 \cdot \frac{\pi}{6} = \frac{2\pi}{3}$

   (c) $\int \frac{1}{16+x^2}\, dx = \frac{1}{16}\int \frac{1}{1+\left(\frac{x}{4}\right)^2} dx = \frac{1}{4}\arctan\left(\frac{x}{4}\right) + C$

5. (a) $\lim_{x \to +\infty} \arctan x = \frac{\pi}{2}$

   (b) $\lim_{x \to -\infty} \tan^{-1} x = -\frac{\pi}{2}$

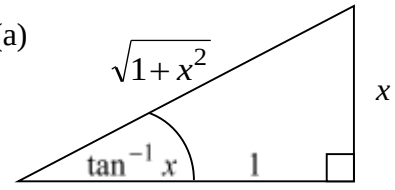
6. (a)



   (b) $\sin\left(\cos^{-1} x\right) = \sqrt{1-x^2}$

   (c) $\frac{d}{dx}\left(\cos^{-1} x\right) = \frac{1}{-\sin\left(\cos^{-1} x\right)} = -\frac{1}{\sqrt{1-x^2}}$

7. (a)



   (b) $\sec\left(\tan^{-1} x\right) = \sqrt{1+x^2}$.

   (c) $\sec^2\left(\tan^{-1} x\right) = 1 + x^2$

   (d) $\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{\sec^2\left(\tan^{-1} x\right)} = \frac{1}{1+x^2}$