Activity 6.4[†‡] – Inverse Trigonometric Functions

FOR DISCUSSION: *State the derivative formulas for arcsine, arccosine, and arctangent.*

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1. Write down each derivative from memory.

   (a) $\dfrac{d}{dx}(\arcsin x) =$

   (b) $\dfrac{d}{dx}\left(\cos^{-1} x\right) =$

   (c) $\dfrac{d}{dx}(\arctan x) =$

2. Evaluate each of the following derivatives.

   (a) $\dfrac{d}{dx}\left(\sin^{-1}\left(e^{x}\right)\right) =$

   (b) $\dfrac{d}{dx}\left(x^{2} \arccos\left(x^{5}\right)\right) =$

   (c) $\dfrac{d}{dx}\left(\dfrac{\tan^{-1}(x)}{\ln x}\right) =$

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† This activity is referenced in Lesson 6.4.

‡ This activity has supplemental exercises.

3. Write down each antiderivative from memory.

(a) $\int \frac{1}{\sqrt{1-x^2}}\,dx =$

(b) $\int \frac{-1}{\sqrt{1-x^2}}\,dx =$

(c) $\int \frac{1}{1+x^2}\,dx =$

4. Evaluate each integral. Use a *u*-substitution or the short cut.

(a) $\int \frac{-1}{\sqrt{1-(3x)^2}}\,dx =$

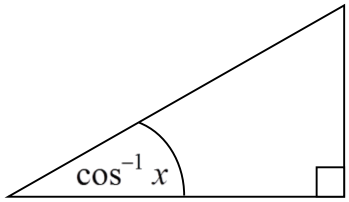
(b) $\int_1^2 \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}\,dx =$

(c) $\int \frac{1}{16+x^2}\,dx =$ (**HINT**: You must first factor out 16 from the denominator.)

5. Write down the following limits from memory.

(a) $\lim_{x \to +\infty} \arctan x =$ (b) $\lim_{x \to -\infty} \tan^{-1} x =$

6. (**OPTIONAL**) (a) By definition, $\cos^{-1} x$ is the angle whose cosine is *x*. Construct a right triangle that satisfies this condition.

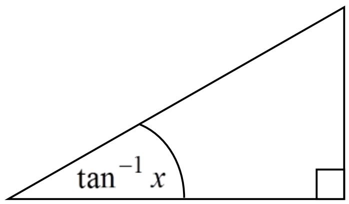


(b) Use the triangle in Part 2(a) to find $\sin\left(\cos^{-1} x\right) =$ ____________________.

(c) Use the derivative-of-an-inverse formula and Part 2(b) to verify the derivative formula for the function $f^{-1}(x) = \cos^{-1} x$.

$$\frac{d}{dx}\left(\cos^{-1} x\right) =$$

7. (**OPTIONAL**) (a) By definition, $\tan^{-1} x$ is the angle whose tangent is *x*. Construct a right triangle that satisfies this condition.



(b) Use the triangle in Part 3(a) to find $\sec\left(\tan^{-1} x\right) =$ ____________________.

(c) Use Part 3(b) to find $\sec^2\left(\tan^{-1} x\right) =$ ____________________.

(d) Use the derivative-of-an-inverse formula and Part 3(c) to verify the derivative formula for the function $f^{-1}(x) = \tan^{-1} x$.

$$\frac{d}{dx}\left(\tan^{-1} x\right) =$$