Examples 6.3 – Other Trigonometric Functions

1. Compute the first and second derivatives of the following functions.

   (a) $f(x) = e^{\tan x}$ (b) $g(x) = \ln(\sec x)$

   Solution: (a) $f'(x) = e^{\tan x} \cdot \sec^2 x$

   $$f''(x) = \left(e^{\tan x} \cdot \sec^2 x\right)\left(\sec^2 x\right) + \left(e^{\tan x}\right)\left(2\sec x \cdot \sec x \tan x\right)$$
   $$= e^{\tan x} \cdot \sec^2 x \cdot \left(\sec^2 x + 2\tan x\right)$$

   (b) $g'(x) = \dfrac{\sec x \tan x}{\sec x} = \tan x$

   $g''(x) = \sec^2 x$

2. Evaluate $\displaystyle\int \frac{\tan x \sec x}{\sin x}\, dx$.

   Solution: $\displaystyle\int \frac{\tan x \sec x}{\sin x}\, dx = \int \frac{\frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}}{\sin x}\, dx = \int \frac{1}{\cos^2 x}\, dx = \int \sec^2 x\, dx = \tan x + C$

3. Consider $\int f(kx+b)dx$. We can use the method of *u*-substitution from Lesson 5.2 by letting $u = kx + b$, so that $du = kdx$ and $dx = \frac{1}{k}du$. It follows that

   $$\int f(kx+b)dx = \int f(u) \cdot \tfrac{1}{k} du = \tfrac{1}{k}\int f(u)du = \tfrac{1}{k}F(u) + C = \tfrac{1}{k}F(kx+b) + C.$$

   In words, if we are integrating a composition in which the inside is the linear function $kx + b$, then we pick up a factor of $1/k$. Use this fact to integrate the following.

   (a) $\int \cos(1.52x - 2.339)\, dx$ (b) $\int \sec^2(1 - 3\theta)\, d\theta$ (c) $\displaystyle\int \frac{1}{5t-4}\, dt$

   Solution: (a) $\int \cos(1.52x - 2.339)dx = \frac{1}{1.52}\sin(1.52x - 2.339) + C$

   (b) $\int \sec^2(1-3\theta)\, d\theta = -\frac{1}{3}\tan(1-3\theta) + C$

   (c) $\displaystyle\int \frac{1}{5t-4}\, dt = \tfrac{1}{5}\ln|5t-4| + C$