Activity 6.3 – Other Trigonometric Functions

1. (a) $\tan x = \frac{\sin x}{\cos x}$

   (b) $\sec x = \frac{1}{\cos x}$

   (c) $\cot x = \frac{\cos x}{\sin x}$

   (d) $\csc x = \frac{1}{\sin x}$

2. (a) $\frac{d}{dx}(\tan x) = \sec^2 x$

   (b) $\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$

   (c) $\frac{d}{dx}(\cot x) = -\csc^2 x$

   (d) $\frac{d}{dx}(\csc x) = -\csc x \cdot \cot x$

3. (a) $\frac{d}{dx}\left(e^{\sec x}\right) = e^{\sec x} \cdot \sec x \cdot \tan x$

   (b) $\frac{d}{dt}(\tan(\ln t)) = \sec^2(\ln t) \cdot \frac{1}{t}$

   (c) $\frac{d}{d\theta}(\theta \cot(2\theta)) = (1)(\cot(2\theta)) + (\theta)\left(-\csc^2(2\theta) \cdot 2\right)$

   (d) $\frac{d}{dx}\left(\tan^2 x\right) = 2\tan x \cdot \sec^2 x$

4. (a) $\int \sec x \tan x \, dx = \sec x + C$

   (b) $\int \sec^2 x \, dx = \tan x + C$

   (c) $\int \csc^2 x \, dx = -\cot x + C$

   (d) $\int \csc x \cot x \, dx = -\csc x + C$

5. (a) $\int \sec^2(3x+2)\, dx = \frac{1}{3}\tan(3x+2) + C$

   (b) $\int \frac{x^2 \sin x - x}{x^2}\, dx = \int \left(\sin x - \frac{1}{x}\right) dx = -\cos x - \ln|x| + C$

6. $\lim_{\theta \to 0} \frac{7\sin\theta}{4\tan\theta + \theta} \overset{LR}{=} \lim_{\theta \to 0} \frac{7\cos\theta}{4\sec^2\theta + 1} = \frac{7}{5}$

7. (a) $\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$

   (b) $\frac{d}{dx}(\cot x) = \frac{d}{dx}\left(\frac{\cos x}{\sin x}\right) = \frac{(-\sin x)(\sin x) - (\cos x)(\cos x)}{\sin^2 x} = -\left(\frac{\sin^2 x + \cos^2 x}{\sin^2 x}\right) = -\left(\frac{1}{\sin^2 x}\right) = -\csc^2 x$

   (c) $\frac{d}{dx}(\sec x) = \frac{d}{dx}\left((\cos x)^{-1}\right) = (-1)(\cos x)^{-2} \cdot (-\sin x) = \frac{\sin x}{\cos^2 x} = \left(\frac{1}{\cos x}\right)\left(\frac{\sin x}{\cos x}\right) = \sec x \tan x$

   (d) $\frac{d}{dx}(\csc x) = \frac{d}{dx}\left((\sin x)^{-1}\right) = (-1)(\sin x)^{-2} \cdot (\cos x) = -\frac{\cos x}{\sin^2 x} = -\left(\frac{1}{\sin x}\right)\left(\frac{\cos x}{\sin x}\right) = -\csc x \cot x$