Examples 6.2 – Derivatives and Antiderivatives of Cosine and Sine

1. Compute the following derivatives.

   (a) $\frac{d}{dx}\left(\sin\left(e^{5x}\right)\right)$ (b) $\frac{d}{dt}\left(-5.21\cos(3t-1.33)\right)$ (c) $\frac{d}{dx}\left(\sin x\cos x\right)$

   Solution: (a) $\frac{d}{dx}\left(\sin\left(e^{5x}\right)\right)=\cos\left(e^{5x}\right)\cdot 5e^{5x}=5e^{5x}\cos\left(e^{5x}\right)$

   (b) $\frac{d}{dt}\left(-5.21\cos(3t-1.33)\right)=5.21\sin(3t-1.33)\cdot 3=15.63\sin(3t-1.33)$

   (c) $\frac{d}{dx}\left(\sin x\cos x\right)=(\cos x)(\cos x)+(\sin x)(-\sin x)=\cos^2 x-\sin^2 x$

2. Evaluate $\lim\limits_{x\to 0}\dfrac{\cos x+3x-1}{\sin x}$.

   Solution: Direct substitution yields the indeterminate form 0/0, so we will apply L'Hôpital's Rule:

$$\lim_{x\to 0}\frac{\cos x+3x-1}{\sin x}\overset{LR}{=}\lim_{x\to 0}\frac{-\sin x+3}{\cos x}=3$$

3. Assuming that the FTC holds for sine and cosine, evaluate $\int_0^{\pi}(5\sin x+2\cos x)\,dx$.

   Solution:
$$\begin{aligned}\int_0^{\pi}(5\sin x+2\cos x)\,dx&=(-5\cos x+2\sin x)\Big|_0^{\pi}\\&=(-5\cos\pi+2\sin\pi)-(-5\cos 0+2\sin 0)\\&=10\end{aligned}$$

4. In Lesson 5.2, we learned via *u*-substitution that $\int e^{kx}dx=\frac{1}{k}e^{kx}+C$. That is, when the "inside" of an exponential is a constant multiple of *x*, then we "pick up" a factor of 1/*k* when integrating. The same is true for $y=\sin kx$ and $y=\cos kx$. Use this fact to evaluate the following.

   (a) $\int\sin 10x\,dx$ (b) $\int 3\cos 2x\,dx$

   Solution: (a) $\int\sin 10x\,dx=-\frac{1}{10}\cos 10x+C$

   (b) $\int 3\cos 2x\,dx=\frac{3}{2}\sin 2x+C$