Activity 6.2 – Derivatives and Antiderivatives of Cosine and Sine

1. (a) $\frac{d}{dx}(\sin x) = \cos x$

   (b) $\frac{d}{dx}(\cos x) = -\sin x$

   (c) $\frac{d}{dx}(\sin 10x) = \cos 10x \cdot 10 = 10\cos 10x$

   (d) $\frac{d}{dx}\left(\frac{1}{2}\cos \pi x\right) = -\frac{1}{2}\sin \pi x \cdot \pi = -\frac{\pi}{2}\sin \pi x$

   (e) $\frac{d}{dx}\left(\sin x^3\right) = \cos x^3 \cdot 3x^2 = 3x^2 \cos x^3$

   (f) $\frac{d}{dx}\left(\cos^3 x\right) = 3\cos^2 x \cdot (-\sin x) = -3\sin x \cos^2 x$

   (g) $\frac{d}{dt}\left(t\sin\left(5t^2\right)\right) = (1)\left(\sin\left(5t^2\right)\right) + (t)\left(\cos\left(5t^2\right)\cdot 10t\right) = \sin\left(5t^2\right) + 10t^2\cos\left(5t^2\right)$

   (h) $\frac{d}{d\theta}\left(\frac{e^{5\theta}}{\sin(2\theta)}\right) = \frac{\left(5e^{5\theta}\right)(\sin(2\theta)) - \left(e^{5\theta}\right)(2\cos(2\theta))}{\sin^2(2\theta)}$

   (i) $\frac{d}{du}(\ln(\cos(3u))) = \frac{1}{\cos(3u)}\cdot(-3\sin(3u)) = \frac{-3\sin(3u)}{\cos(3u)}$

2. (a) $\int \cos x\, dx = \sin x + C$

   (b) $\int \sin x\, dx = -\cos x + C$

   (c) $\int 2\cos(3t)\, dt = \frac{2}{3}\sin(3t) + C$

   (d) $\int 0.5\cos\left(\frac{x}{4}\right)dx = 2\sin\left(\frac{x}{4}\right) + C$

   (e) $\int_0^2 \sin\left(\frac{\pi}{2}x\right)dx = \left(-\frac{2}{\pi}\cos\left(\frac{\pi}{2}x\right)\right)\Big|_0^2 = \frac{4}{\pi}$

3. (a) $\lim_{x\to 0}\frac{\sin(5x)}{\sin(8x)} \overset{LR}{=} \lim_{x\to 0}\frac{5\cos(5x)}{8\cos(8x)} = \frac{5}{8}$

   (b) $\lim_{x\to 2}\frac{\sin(2x-4)}{\cos(\pi x) - \frac{x}{2}} \overset{LR}{=} \lim_{x\to 2}\frac{2\cos(2x-4)}{-\pi\sin(\pi x) - \frac{1}{2}} = -4$

4. (a) $h(23) = 1.18\cos\left(\frac{2\pi}{11}(23)\right) + 2.78 \approx 3.77$ meters

   (b) $h'(t) = (1.18)\cdot\left(\frac{2\pi}{11}\right)\cdot\left(-\sin\left(\frac{2\pi}{11}t\right)\right) = -\frac{2.36\pi}{11}\sin\left(\frac{2\pi}{11}t\right)$ meters per hour, where t is hours after 9:00 a.m. on July 22.

   (c) $h'(23) = -\frac{2.36\pi}{11}\sin\left(\frac{2\pi}{11}(23)\right) \approx -0.36$ meters per hour; negative rate means tide is falling.

5. $y' = \frac{3-\cos x}{4+\sin y}$