Activity 6.2‡ – Derivatives and Antiderivatives of Cosine and Sine

FOR DISCUSSION: *State the derivative formulas for the cosine and sine functions.*
State the indefinite integral formulas for the cosine and sine functions.

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1. Compute each derivative.

(a) $\frac{d}{dx}(\sin x) =$

(b) $\frac{d}{dx}(\cos x) =$

(c) $\frac{d}{dx}(\sin 10x) =$

(d) $\frac{d}{dx}\left(\frac{1}{2}\cos \pi x\right) =$

(e) $\frac{d}{dx}\left(\sin x^3\right) =$

(f) $\frac{d}{dx}\left(\cos^3 x\right) =$

(g) $\frac{d}{dt}\left(t \sin\left(5t^2\right)\right) =$

(h) $\frac{d}{d\theta}\left(\frac{e^{5\theta}}{\sin(2\theta)}\right) =$

(i) $\frac{d}{du}(\ln(\cos(3u))) =$

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‡ This activity has supplemental exercises.

2. Compute each integral.

(a) $\int \cos x \, dx =$

(b) $\int \sin x \, dx =$

(c) $\int 2\cos(3t) dt =$

(d) $\int 0.5\cos\left(\frac{x}{4}\right) dx =$

(e) $\int_0^2 \sin\left(\frac{\pi}{2} x\right) dx =$

3. Compute the following limits.

(a) $\lim_{x \to 0} \frac{\sin(5x)}{\sin(8x)} =$

(b) $\lim_{x \to 2} \frac{\sin(2x-4)}{\cos(\pi x) - \frac{x}{2}} =$

4. In Activity 6.1 we considered the following problem:

   Suppose that you were sitting on a pier at your favorite ocean resort on July 22 at 9:00 a.m., just as high tide was occurring. A nearby tide gauge showed the water level at 3.96 meters. Assuming that the interval between high and low tides at this resort is 5.5 hours, you decided to return to the pier at 2:30 p.m. on that same day to check the water level and observed it to be 1.60 meters. Write an equation of the form $h(t) = A\cos(Bt) + D$ that models the height of the tide at the resort t hours after 9:00 a.m. on July 22 (i.e., $t = 0$ corresponds to 9:00 a.m.).

   The solution to the problem is $h(t) = 1.18\cos\left(\frac{2\pi}{11}t\right) + 2.78$ meters, where t is hours after 9:00 a.m. on July 22.

   (a) What was the height of the tide at 8:00 a.m. on July 23?

   (b) Find the rate of change formula for the height of the tide. Include units in your answer.

   (c) Was the tide rising or falling at 8:00 a.m. on July 23? How do you know?

5. Use implicit differentiation to find y' if $\sin x - \cos y = 3x - 4y$.