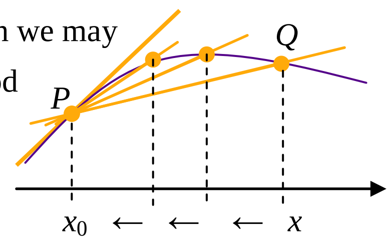
# Lesson 2.1 – Derivatives of Quadratic Functions

Informally, a **tangent line** to the graph of a function $f$ at a point $P = (x_0, f(x_0))$ is a line that intersects the graph at *P*, and "points in the same direction" as the graph does at *P*. We define the derivative $f'(x_0)$ to be the slope of the tangent line at $x = x_0$. Two points on a line define its slope, but *P* is the only point we know of on the tangent line. If we try to use *P* for both points in the average rate of change formula, then $x = x_0$, and $\frac{\Delta y}{\Delta x} = \frac{f(x_0) - f(x_0)}{x_0 - x_0} = \frac{0}{0}$, which is a so-called **indeterminate form** (to be discussed later). This does not necessarily mean that the slope of the tangent line is undefined, but it does mean that the slope cannot be determined by examining the behavior of the graph *at* the point *P*. The problem is the $x - x_0$ in the denominator.

Another option is to analyze the behavior of the curve *near P*. If we choose an $x$ near $x_0$, then $x - x_0 \neq 0$, and the slope of the secant line through *P* and $Q = (x, f(x))$ *approximates* the slope of the tangent line at *P*. If we choose $x$ closer and closer to $x_0$, then we may be able to find better and better approximations. In practice, a good approximation is sometimes all we need, but in theory, we'd like an exact answer. For certain functions (like quadratics), we can first use algebra to eliminate the denominator of the secant slope formula, and then plug in $x_0$ without any trouble.




We have outlined above a three-step method that uses slopes of secant lines to "sneak up" on the slope of the tangent line to $f$ at $P = (x_0, f(x_0))$:

STEP 1: Pick $x \neq x_0$.

STEP 2: Simplify the secant slope $\frac{\Delta y}{\Delta x} = \frac{f(x) - f(x_0)}{x - x_0}$ and eliminate the denominator.

STEP 3: Let $x$ get closer and closer to $x_0$ from either side, denoted $x \to x_0$. (For now, plug in $x_0$ for $x$.) The result, if one exists, is $f'(x_0)$. We will discuss existence later.

Now consider the **quadratic function** $f(x) = ax^2 + bx + c$. Let $P = (x_0, f(x_0))$ be a point on the graph, and suppose we want the derivative $f'(x_0)$. If $x \neq x_0$, then the average rate of change is

$$\frac{f(x) - f(x_0)}{x - x_0} = \frac{(ax^2 + bx + c) - (ax_0^2 + bx_0 + c)}{x - x_0} = \frac{a(x + x_0)(x - x_0) + b(x - x_0)}{x - x_0} = a(x + x_0) + b$$

The denominator cancelled, which allows us to plug in $x_0$ for $x$ to get $f'(x_0) = 2ax_0 + b$. In other words, the derivative formula of a quadratic is linear:

$$\text{If } f(x) = ax^2 + bx + c, \text{ then } f'(x) = 2ax + b.$$