Homework 4.1 – Analyzing Rational Functions

1. Consider the expression \( \frac{x^2}{x-3} - \frac{4}{x-4} \).

(a) Combine the two terms and enter the result in reduced form as a ratio of polynomials by eliminating all parentheses. (Hint: Find a common denominator.)

\[ \frac{x^2}{x-3} - \frac{4}{x-4} = \frac{x^2 - 4}{x-3} \]

(b) Use long division to rewrite the answer to part (a) in proper form, \( Q(x) + \frac{R(x)}{D(x)} \), where \( D(x) = x^2 - 7x + 12 \).

2. If \( f(x) = \frac{\sqrt{x} - 7}{\sqrt{x} + 7} \), then by the quotient rule,

\[ f'(x) = \text{___________} \]

3. A drug is injected into the bloodstream of a patient. The concentration (in milligrams per cubic centimeter) of the drug in the bloodstream \( t \) hours after the injection is given by

\[ C(t) = \frac{0.16t}{t^2 + 6}. \]

(a) By the quotient rule, the rate of change of the drug concentration with respect to time after a half hour is

\[ \text{___________} \text{ mg per cubic cm per hr.} \]

(b) How fast is the drug concentration changing after 3 hours?

\[ \text{___________} \text{ mg per cubic cm per hr} \]

4. The pressure on an object \( P \) in Pascals (Pa) is proportional to the temperature \( T \) in degrees Celsius (C) and inversely proportional to the volume \( V \) in cubic meters (m³), where all three quantities depend on time \( t \) in seconds (s). That is,

\[ P(t) = \frac{T(t)}{V(t)} \]

At time \( t = 8 \) seconds, the temperature of the object is 66C and changing at a rate of \( \frac{2C}{s} \). At this same time, the volume is 5m³ and changing at the rate \( -1 \frac{m^3}{s} \). By the quotient rule, the pressure on the object at \( t = 8 \) seconds is changing at the rate of \( P'(8) = \text{___________} \text{ units} \).

5. Let \( f(x) = \frac{x^2 - (13)x + 42}{x^2 + 5x + 6} \).

(a) The \( y \)-intercept is \( y = \text{___________} \).

(b) The \( x \)-intercept(s) is/are \( x = \text{___________} \).

(c) The vertical asymptote(s) is/are \( x = \text{___________} \).

(d) To find the local extrema of \( f \), we set \( f'(x) = 0 \) and solve for \( x \). In this case, \( f'(x) = \text{___________} \), and we must find the zeros of the numerator. In reduced form (expanded without parentheses), we must ultimately solve the equation

\[ \text{___________} = 0. \]

Therefore, the local extrema are at \( x = \text{___________} \).

(e) Use the information from parts (a)-(d) (and without a calculator!) to choose the correct graph of \( f \).