Examples 1.4 – Integrals of Constant Functions

1. Evaluate each of the following.

   (a) \((9x - 12)’\)  \hspace{1cm} (b) \(\int 9 \, dt\)  \hspace{1cm} (c) \((2.991 - 0.423u)’\)  \hspace{1cm} (d) \(\int (-0.423) \, dT\)

   **Solution:** (a) The slope of the linear function \(y = 9x - 12\) is \((9x - 12)’ = 9\).

   (b) The family of linear functions that have slope \(m = 9\) is \(\int 9 \, dt = 9t + C\).

   (c) The slope of the linear function \(y = 2.991 - 0.423u\) is \((2.991 - 0.423u)’ = -0.423\).

   (d) The family of functions that have slope \(m = -0.423\) is \(\int (-0.423) \, dT = -0.423T + C\).

2. If a savings account increases by $110 per month, then how much money is saved from month 5 to month 10?

   **Solution:** We must compute the net change in the amount in the account over \([5, 10]\) given that the rate of increase is $110 per month. To do so, we need a member of the family

   \[\int 110 \, dt = 110t + C\]

   so we simply choose the one with \(C = 0\). Therefore, by the Fundamental Theorem, we have

   \[\int_5^{10} 110 \, dt = (110t)|_5^{10} = 110(10) - 110(5) = 550 \text{ dollars.}\]

   (We could have computed the answer algebraically by multiplying $110 by 5 months, but only because the rate is constant. We will see that the FTC holds for variable rates as well.)

3. Evaluate \(\int_{-3}^{6} (-4) \, dx\) and sketch the geometrical interpretation of the answer.

   **Solution:** By the Fundamental Theorem, \(\int_{-3}^{6} (-4) \, dx = (-4x)|_{-3}^{6} = (-4)(6) - (-4)(-3) = -36\), which represents the net signed area between the graph of \(y = -4\) and the \(x\)-axis on \([-3, 6]\):

   ![Graph](image-url)