Examples 1.3 – Derivatives of Linear Functions

1. Find the first and second derivatives of \( y = 4x + 1 \), \( g(t) = 3 - 5t \), and \( h(r) = 1.344 \).

**Solution:** Since all three of the given functions are linear, the derivative of each function is simply its slope. That is, \( y' = 4 \), \( g'(t) = -5 \), and \( h'(r) = 0 \). For the same reason, the second derivatives are \( y'' = 0 \), \( g''(t) = 0 \), and \( h''(r) = 0 \).

2. The rate of change of the position over time of a moving object is its velocity \( v(t) \), and the rate of change of velocity over time is its acceleration \( a(t) \). If the position of an object after \( t \) minutes is given by \( s(t) = 65t + 20 \) cm, then what are its velocity and acceleration functions?

**Solution:** In general, if \( s(t) = 65t + 20 \), then \( s'(t) = 65 \) and \( s''(t) = 0 \). We must express our answers in the context of the problem with appropriate units, and the words “over time” give us a hint as to how to do this: If \( s(t) = 65t + 20 \) cm, then the velocity \( v(t) = s'(t) = 65 \) cm/min (centimeters per minute). If \( v(t) = s'(t) = 65 \) cm per minute, then the acceleration function is \( a(t) = v'(t) = s''(t) = 0 \) cm per minute per minute, or cm/min\(^2\).

4. For each part, sketch an example of a (possibly nonlinear) graph having the given properties.

   (i) A constant derivative of two.
   (ii) A negative derivative at \( x = 1 \), and a positive derivative at \( x = 3 \).
   (iii) A zero derivative at \( x = -1 \), positive derivatives on the interval \((-1, 2)\), and a zero derivative at \( x = 2 \).

**Solution:** There are many correct solutions. Here are some possibilities.