AU

William Varick Nevins III High School Mathematics Competition
Fall 2012

Division of Mathematics
Alfred University
Alfred, NY 14802

Instructions:

1. This competition will last seventy-five minutes – from 10:05 to 11:20.

2. Put your five-digit student number in the correct place on the computer answer sheet.

3. The use of calculators is not permitted on this examination.

4. There are thirty questions. Mark your answers on the computer answer sheet. Use a #2 pencil only. You may use this question booklet for scratch work.

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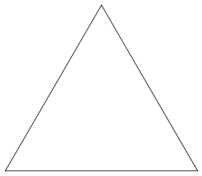
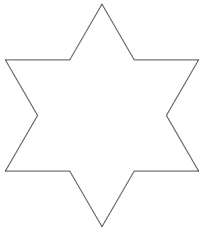
1. An equilateral triangle with side length 20 is drawn. On each side of the triangle the middle third of the side is removed, and another two lines with length equal to removed part are drawn as shown in the picture. What is the perimeter of the resulting figure?





A) 60

B) $60\frac{\sqrt{5}}{2}$

C) $40\frac{\sqrt{3}+1}{\sqrt{3}}$

D) 80

E) $60\sqrt{3}$

2. Find the value of x if $\frac{3}{5} = \frac{1}{1+\frac{1}{1+\frac{1}{x}}}$.

A) -2

B) -1

C) 0

D) 1

E) 2

3. Often in buying a product at a supermarket, there is a concern about the item being underweight. Suppose a local grocery store has 12 "one pound" packages of frozen ground turkey on display, 2 of which are underweight. If you are going to buy the fourth package sold that day at random, what is the probability that the one you selected is an underweight one?

   A) 1/7

   B) 1/6

   C) 1/5

   D) 1/4

   E) 1/3

4. A new kind of chess piece is placed in the upper left hand corner of a 5x5 chessboard. This piece will move 3 squares horizontally or vertically followed by one block in the perpendicular direction. Space permitting, at each move the piece can move three squares left or right followed by one unit up or down or it can move three squares up or down followed by one unit left or right. The first two possible moves for the piece are illustrated below. How many squares can the piece not get to?

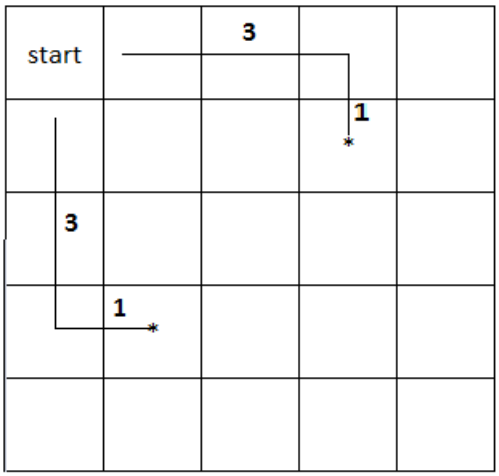
   A) 0

   B) 5

   C) 8

   D) 14

   E) 16



5. Solve for x: $(x^2 - 3x + 3)^{x^2-3x} = 1$.

   A) $x = 0$

   B) $x = 0$ or $x = 1$

   C) $x = 0$, $x = 2$ or $x = 1$

   D) $x = 0$, $x = 1$, $x = 2$ or $x = 3$

   E) $x = 0$, $x = 1$, $x = 2$, $x = 3$ or $x = 4$

6. There are ten lines in the plane. Three are parallel to each other and five are parallel to each other but not to the first three. The remaining lines are not parallel to any others. No three lines intersect in a common point. How many intersections are there?

   A) 15

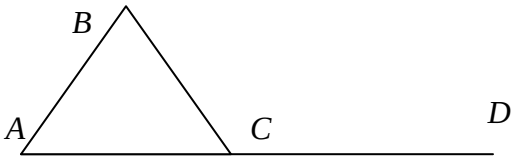
   B) 31

   C) 32

   D) 33

   D) 42

7. In moving in the graph below, moving from one vertex to another adjacent vertex is counted as one step. If you may trace a line more than once and in either direction, in how many ways can you move from *A* to *C* in three steps?



A) 2

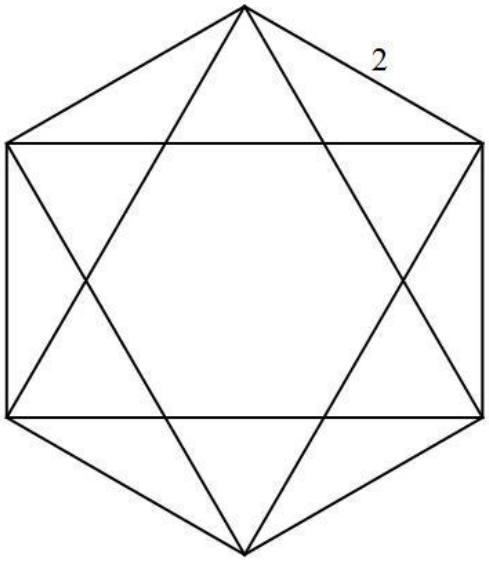
B) 3

C) 4

D) 5

E) 6

8. A hexagon with 6 equal sides of length 2 is drawn. Each vertex of the hexagon is connected to two of the other vertices of the hexagon by a straight line, forming a smaller interior hexagon with equal sides. How long are the sides of the interior hexagon?



A) $\frac{1}{2}$

B) $\frac{1}{\sqrt{2}}$

C) 1

D) $\frac{2}{\sqrt{3}}$

E) $\sqrt{3}$

9. Point A' lies 90% of the way from *A* to *B* on square *ABCD*. Similarly,
   B' lies 90% of the way from *B* to *C*,
   C' lies 90% of the way from *C* to *D*,
   D' lies 90% of the way from *D* to *A*,
   Find the ratio of the area of quadrilateral $A'B'C'D'$ to *ABCD*.

   A) .72

   B) .81

   C) .82

   D) .9

   E) .91

10. Let $P(x) = ax^2 + bx + c$, where *a*, *b*, and *c* are constants. What is the value of b if $P(x+1) - P(x) = x$ for all *x*?

    A) -1

    B) $-\frac{1}{2}$

    C) 0

    D) $\frac{1}{2}$

    E) 1

11. We choose at random a whole number from 1 to 10000. What is the probability that the number is divisible by 3 or divisible by 7 or by both?

    A) 4761/10000

    B) 4662/10000

    C) 4527/10000

    D) 4384/10000

    E) 4285/10000

12. How many zeros are at the end of 2012! (where $n! = n(n-1)(n-2)\ldots(2)(1)$)?

A) 101

B) 402

C) 482

D) 500

E) 501

13. If 4A67 is a four-digit number divisible by 11, and 45B7 is a four-digit number divisible by 9, then what is A + B?

A) 3

B) 5

C) 7

D) 9

E) 11

14. $1 + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15} + \frac{1}{21} + \frac{1}{28} + \frac{1}{36} + \frac{1}{45} + \frac{1}{55} =$

A) $\frac{10}{11}$

B) $\frac{15}{11}$

C) $\frac{20}{11}$

D) $\frac{25}{11}$

E) $\frac{30}{11}$

15. How many distinct license plates can be made using each of the characters A, U, 2, 0, 1, 2 once and only once.

A) 120

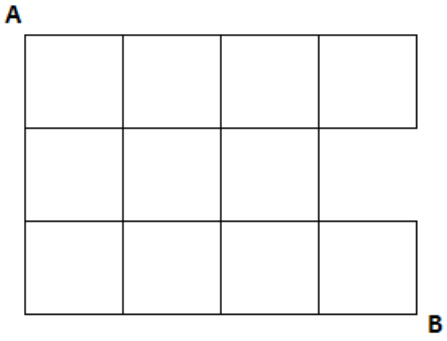
B) 300

C) 360

D) 720

E) 1024

16. The lines on the grid below represent roads on which a salesperson can travel. The salesperson starts at vertex A, they must get to vertex B, and they must always travel in a direction that takes them closer to B. How many routes can the salesperson take?



A) 15

B) 20

C) 25

D) 30

E) 35

17. Consider the first few terms of a sequence: 1,2,4,4,7,8,10,16,13,32,… What is the sum of the next two consecutive terms in this sequence?

A) 41

B) 52

C) 55

D) 78

E) 80

18. Determine the last two digits of 2011^{2012}.

    A) 11

    B) 21

    C) 31

    D) 41

    E) 51

19. Suppose an equilateral triangle is inscribed inside of a unit circle. Then a circle is inscribed inside of that triangle. Also, three small circles are inscribed in the regions made by the edges of the equilateral triangle and the outer unit circle. What is the total area of the sum of the three small circles and the circle inscribed inside the equilateral triangle?

    A) $\pi/32$

    B) $3\pi/16$

    C) $\pi/4$

    D) $5\pi/16$

    E) $7\pi/16$

20. A bucket contains two red balls, one blue ball and one green ball. You reach into the bucket at pull out a ball, replacing it with a red ball regardless of the color you pulled out. What is the probability that all balls in the bucket are red after three such replacements?

    A) 1/8

    B) 9/32

    C) 5/16

    D) 1/2

    E) 7/12

21. What is the value of $\sqrt[3]{2+\sqrt{5}}+\sqrt[3]{2-\sqrt{5}}$?

A) $\sqrt{1}$

B) $\sqrt{2}$

C) $\sqrt{3}$

D) $\sqrt{4}$

E) $\sqrt{5}$

22. How many triples of the form (x, y, z) satisfy the set of equations $xy = z$, $xz = y$, and $yz = x$?

A) 2

B) 5

C) 6

D) 8

E) 9

23. How many integers n in the set $\{1 , \ldots , 100\}$ have the property that $n^2 + n^3$ is the square of an integer?

A) 1

B) 3

C) 5

D) 7

E) 9

24. Which number is the largest in magnitude?

A) 2^{11000}

B) 3^{7000}

C) 5^{5000}

D) 7^{3000}

E) 11^{2000}

25. Suppose f is a polynomial with integer coefficients such that $f(a) = a$ and $f(0) = p$ for some prime $p > a$. Find a.

A) 0

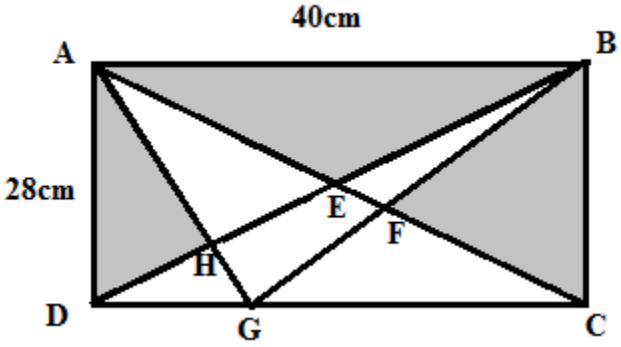
B) 1

C) 2

D) 3

E) 4

26. The length and the width of the rectangle are 40cm and 28cm in the following graph. The area of the shaded region is 580cm^2. The area of the region EFGH is



A) $10\ \text{cm}^2$

B) $15\ \text{cm}^2$

C) $20\ \text{cm}^2$

D) $25\ \text{cm}^2$

E) $30\ \text{cm}^2$

27. Let $f(x) = x^4 + x^3 + Ax^2 + Bx + 1$. The remainder after dividing f by $x + 1$ is 5, and the remainder after dividing f by $x - 1$ is 7. What is the remainder after dividing f by $x - 2$?

A) 41

B) 43

C) 45

D) 47

E) 49

28. We have two three digit positive integers **abc** and **def**. If we put **abc** in the front of **def** we form a 6 digit integer which is 6 times of the integer by putting **def** in the front of **abc**. We also know the sum of these two integers is 999. Then the difference of these two integers is

    A) 325

    B) 436

    C) 547

    D) 628

    E) 715

29. Peter drove from city A to city B. If he increases his average speed by 20%, then he will arrive city B an hour in advance. If he drove the first 120 miles at the same average speed and increases his average speed by 20% in the rest of the time, then he will arrive city B 40 minutes in advance. The distance between city A and City B is

    A) 240 miles

    B) 360 miles

    C) 480 miles

    D) 600 miles

    E) 720 miles

30. A farmer constructs a barn in the shape of a regular dodecagon (12 equal length sides, 12 equal angles) where each side has length 20 ft. He ties a goat to a corner of the barn with a rope of length 40ft. What is the area of grass that the goat can eat from?

A) $\frac{3200}{3}\pi$

B) 1000π

C) 1400π

D) 1500π

E) 1600π