AU

William Varick Nevins III High School Mathematics Competition
Fall 2011

Division of Mathematics
Alfred University
Alfred, NY 14802

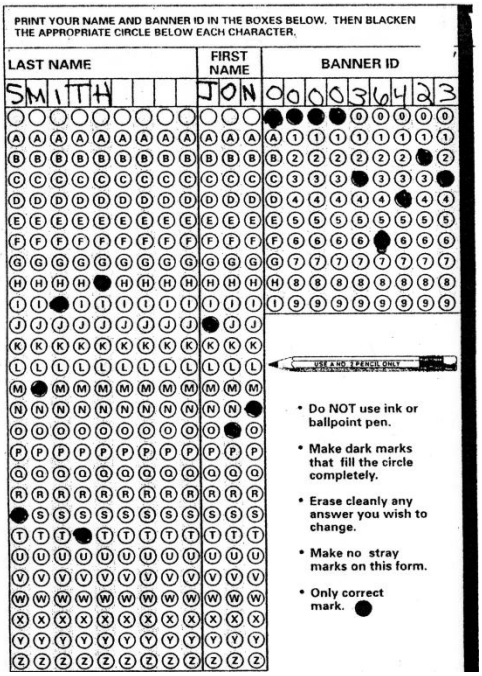
Instructions:

1. This competition will last seventy-five minutes – from 10:05 to 11:20.

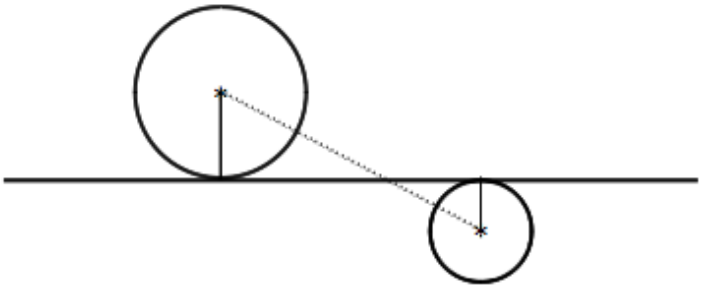
2. The use of calculators is not permitted.

3. There are thirty questions. Mark your answers on the computer answer sheet. Use a #2 pencil only. You may use this question booklet for scratch work. You may keep this booklet.

4. Put your name on the Scantron. Your last name must start in the left column of the last name section. You do not need to put blanks at the end of your name. Put your nine-digit student number in the Banner ID on the Scantron sheet. You **must** include the three zeros at the beginning of this number. If your name and number are not on the sheet your answers will not be graded.



1. A circle of radius 3 and a circle of radius 5 are tangent to the same line, as shown in the figure below.



If the distance between the points of tangency is 15, then what is the distance between the centers of the circles?

A) 16

B) 17

C) 20

D) 22

E) 25

2. The digits 1 through 6 are represented by the letters *a* through *f*, although not necessarily in that order. Determine the two-digit number *ab* if

$$ab - c - d - e - f = 16.$$

A) 22

B) 29

C) 32

D) 39

E) 42

3. Which of the following amounts cannot be obtained with exactly five American coins?

   A) 25 cents

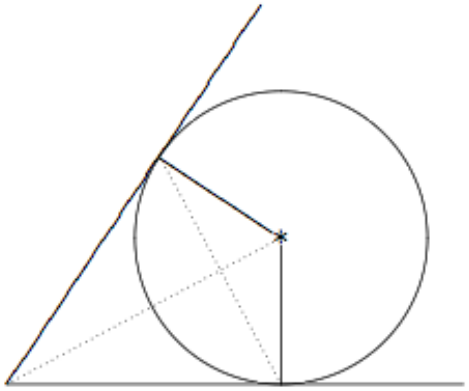
   B) 26 cents

   C) 27 cents

   D) 28 cents

   E) 29 cents

4. Two lines are drawn tangent to a circle of radius 8, as shown in the figure below below.



   If the lines intersect 17 units away from the center of the circle, then what is the length of the chord connecting the points of tangency?

   A) 14

   B) $\frac{240}{17}$

   C) 15

   D) $\frac{255}{8}$

   E) 255

5. While watching the circus performance, Bill decided to count the number of unicycles, bicycles, and tricycles. He counted a total of 21 cycles, 2 unicycles, and 45 wheels. How many bicycles did Bill count?

   A) 3

   B) 5

   C) 8

   D) 11

   E) 14

6. For positive integers *n*, the first 2011 terms of the sequence $\frac{2^n}{n}$ are $\frac{2}{1}, \frac{4}{2}, \frac{8}{3}, \ldots, \frac{2^{2011}}{2011}$. How many of these terms are whole numbers?

   A) 11

   B) 21

   C) 31

   D) 41

   E) 51

7. In how many ways could we arrange two orange flags, three red flags, and two blue flags in a row, assuming that like-colored flags are indistinguishable?

   A) 100

   B) 120

   C) 150

   D) 180

   E) 210

8. Determine the last two digits of 2011^{2011}.

   A) 01

   B) 11

   C) 21

   D) 31

   E) 41

9. Inscribed in the rectangle below are three mutually tangent circles, each with radius 1 foot. What is the area of the rectangle?

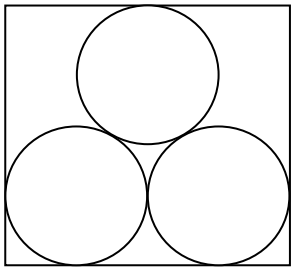
   A) 4 ft^2

   B) $4 + 2\sqrt{3}$ ft^2

   C) 16 ft^2

   D) 3π ft^2

   E) $8 + 4\sqrt{3}$ ft^2



10. Two coins of different sizes are placed side by side, and the smaller coin is revolved around the larger coin with no slippage. How many revolutions of the smaller coin are required to return it to its starting position, given that the radius of the smaller coin is half the radius of the larger?

A) ½

B) 1

C) 2

D) 3

E) 4

11. Find $|FC|$ in the parallelogram below, given that $|AE| = 5$, $|BC| = 20$, and $|EF| = 9$.

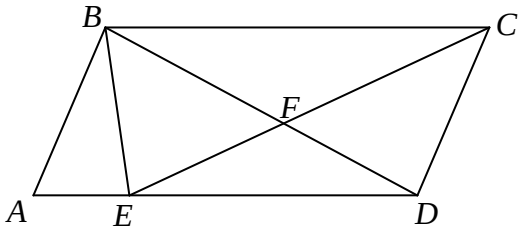
A) 12

B) 13

C) 14

D) 15

E) 16



12. If $x + \frac{1}{x} = 3$ and $x^2 + \frac{1}{x^2} = 7$, then what is $x^4 + \frac{1}{x^4}$?

A) 45

B) 47

C) 49

D) 51

E) 53

13. If $f(x) = ax^6 - bx^2 + 2x + 7$ and $f(-2) = 3$, then find $f(2)$.

A) 3

B) 5

C) 7

D) 9

E) 11

14. Jane needs 6 days to finish a project by herself. Jack needs 10 days to finish the same project by himself. If Jane, Jack, and Peter work on the project together, then they need 2.5 days to finish it. How many days does Peter need to finish the project by himself?

A) 6 days

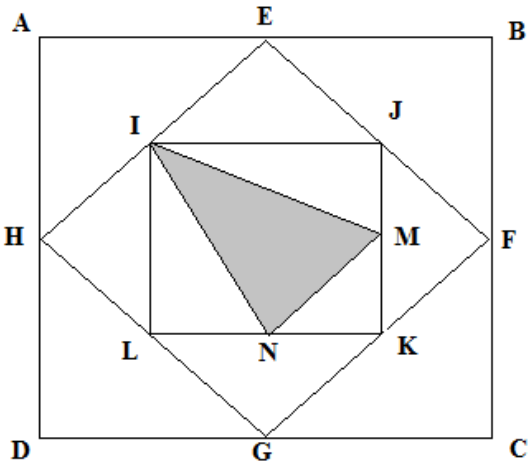
B) 6.5 days

C) 7 days

D) 7.5 days

E) 8 days

15. In the figure below, the points labeled *E*, *F*, *G*, and *H* are the midpoints of the sides of the square *ABCD*. The points labeled *I*, *J*, *K*, and *L* are the midpoints of the sides of the square *EFGH*. The points labeled *M* and *N* are the midpoints of the segments *JK* and *KL*. If the area of the shaded region is 12 cm^2, then find the length of *AB*.



A) 6 cm

B) $6\sqrt{2}$ cm

C) 8 cm

D) $8\sqrt{2}$ cm

E) 10 cm

16. Pocket Pal Incorporated sells its pocket protectors either as single units or in packs of seven. Pocket Pal charges $2.00 for each pack of seven, and $3.00 for each additional protector. According to this pricing scheme, which of the following would cost the most?

A) 24 protectors

B) 33 protectors

C) 39 protectors

D) 50 protectors

E) 70 protectors

17. Let $a > 1$ be an integer and suppose that 70, 155, and 223 each leave the same remainder upon division by *a*. Find the remainder of 2011 upon division by *a*.

A) 1

B) 2

C) 3

D) 4

E) 5

18. You need 10 liters of a 12% acid solution for a laboratory experiment, but your supplier only ships a 10% solution and a 20% solution. Rather than pay the hefty surcharge to have the supplier make a 12% solution, you decide to mix 10% solution with 20% solution to make your own 12% solution. How many liters of 10% solution and 20% solution should you use to make 10 liters of 12% solution?

A) 2 liters of 10% solution and 8 liters of 20% solution

B) 4 liters of 10% solution and 6 liters of 20% solution

C) 6 liters of 10% solution and 4 liters of 20% solution

D) 8 liters of 10% solution and 2 liters of 20% solution

E) 9 liters of 10% solution and 1 liter of 20% solution

19. Congratulations! You made it to the final round of the game show, *Flipping for Dollars*! Your final challenge is to flip a fair coin 10 times and have each flip result in heads. If you succeed, then you will win 10 million dollars! If your first 9 flips result in heads, then what is the probability that you will win the money?

A) $\frac{1}{1024}$

B) $\frac{1}{10}$

C) $\frac{1}{2}$

D) $\frac{9}{10}$

E) $\frac{1023}{1024}$

20. Let $i = \sqrt{-1}$. Compute the sum $\sum_{k=0}^{2011} i^k$.

A) 0

B) 1

C) –1

D) i

E) $-i$

21. Willie visited three fairs. At the first fair, he doubled his money and then spent \$30. At the second fair, he tripled his money and then spent \$54. At the third fair, he quadrupled his money and then spent \$72. If Willie returned home with \$48, how much money did he have at the start?

A) \$17

B) \$21

C) \$25

D) \$29

E) \$34

22. Given a 45-45-90 triangle with two sides of length 10, fix a vertex *V* where one of these sides meets the hypotenuse. At vertex *V*, construct a 45-45-90 triangle whose hypotenuse is the side of length ten. At the same vertex *V*, construct a 45-45-90 triangle whose hypotenuse is a side of the triangle just constructed. Repeat this process at vertex *V* until no more triangles can be constructed (overlap is not allowed). What is the area of the resulting figure?

A) $\frac{6375}{64}$

B) 100

C) $\frac{6375}{32}$

D) 200

E) $\frac{6375}{16}$

23. Ed, Nick, and Tom each have some peanuts. Ed gives to Nick and Tom enough peanuts to double their totals. Then Nick gives to Ed and Tom enough peanuts to double their totals. Finally, Tom gives to Nick and Ed enough peanuts to double their totals. If each person ends up with sixteen peanuts, how many peanuts did each have at the beginning?

A) Ed = 32, Nick = 8, Tom = 8

B) Ed = 24, Nick = 16, Tom = 8

C) Ed = 16, Nick = 16, Tom = 16

D) Ed = 26, Nick = 14, Tom = 8

E) Ed = 24, Nick = 12, Tom = 12

24. A *palindrome* is a sequence of characters that reads the same forward or backward. For example, kayak and 213312 are palindromes. How many palindromes appear in the sequence of integers from 0 through 2011?

A) 119

B) 120

C) 121

D) 122

E) 123

25. A certain equilateral triangle has an area of 20 cm^2. The midpoints of the sides of the triangle are connected, forming a new triangle in the center of the original. If this process is repeated four more times for each of the new center triangles, then what will be the area of the smallest center triangle?

A) 5/256 cm^2

B) 3/64 cm^2

C) 5/32 cm^2

D) 1/4 cm^2

E) 4 cm^2

26. Tom has blood that will cure a vampire's illness, and he cannot be killed by a vampire bite (no matter how much blood is lost). If a vampire bites him, then the vampire becomes human again and inherits the ability to cure. Tom enters a room filled with 200 bloodthirsty vampires. If two vampires feed on one human at a time, then after how many feedings will the humans first outnumber the vampires?

A) 4

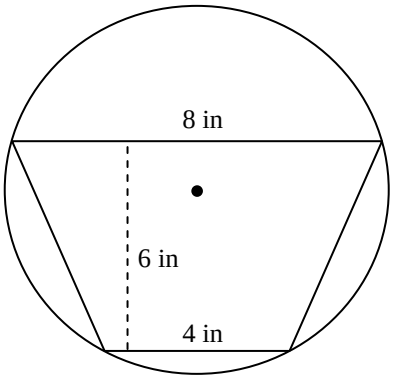
B) 5

C) 6

D) 7

E) 8

27. A trapezoid of height 6 inches is inscribed in a circle. If the lengths of short and long parallel sides of the trapezoid are 4 inches and 8 inches, respectively, then what is the radius of the circle?



A) $\sqrt{16}$ inches

B) $\sqrt{18}$ inches

C) $\sqrt{20}$ inches

D) $\sqrt{22}$ inches

E) $\sqrt{24}$ inches

28. An ancient pyramid with a square base was discovered beneath the permafrost on Antarctica. Estimates currently put its height at 100 ft and its base area at 90000 ft^2. The research crew has uncovered the top quarter of its height, but a snow storm is coming. To protect their find, the archeologists will cover it with an insulated tarp along its sloped faces. How large must this tarp be?

A) $3750\sqrt{13}$ ft^2

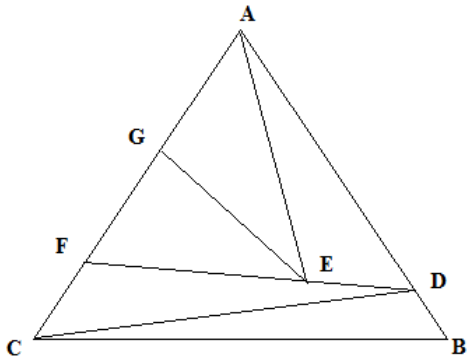
B) 22500 ft^2

C) $1875\sqrt{13}$ ft^2

D) 5625 ft^2

E) 900000 ft^2

29. In the figure below, triangle *ABC* is equilateral, and the areas triangles *AEG*, *AED*, *EGF*, *CFD*, and *CBD* are equal. If AC = 160 cm, then AG =?



A) 30 cm

B) 40 cm

C) 50 cm

D) 60 cm

E) 70 cm

30. A goat is tied with a 40-foot-long rope to the outside corner of a barn. The sides of the barn are 20-feet long and form an equilateral triangle. What is the total area of the region within which the goat can graze? (Note: The goat is not allowed in the barn.)

A) $\frac{2\pi}{3}40^2 + \frac{\pi}{3}20^2$ ft^2

B) $\frac{2\pi}{3}40^2 + \frac{\pi}{3}20^2 + 200$ ft^2

C) $\frac{5\pi}{3}40^2 + \frac{\pi}{3}20^2 + \frac{\pi}{6}10^2$ ft^2

D) $\frac{5\pi}{6}40^2 + \frac{\pi}{3}20^2 + 100\sqrt{3}$ ft^2

E) $\frac{5\pi}{6}40^2 + \frac{\pi}{3}20^2 + \frac{\pi}{3}10^2$ ft^2