AU

William Varick Nevins III High School Mathematics Competition
Fall 2010

Division of Mathematics
Alfred University
Alfred, NY 14802

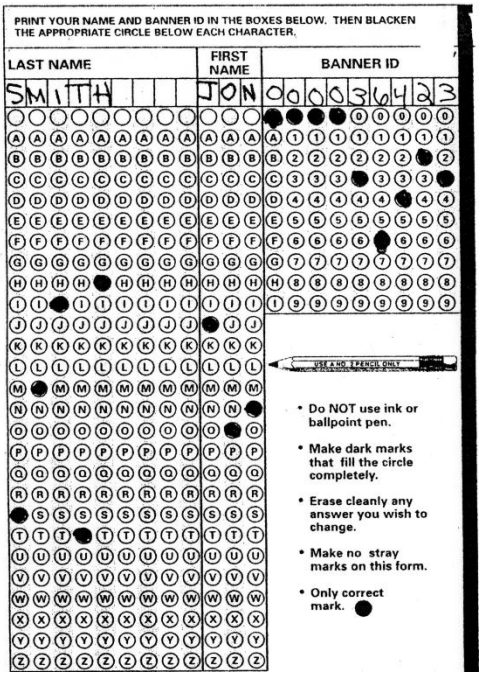
Instructions:

1. This competition will last seventy-five minutes – from 10:05 to 11:20.

2. The use of calculators is not permitted.

3. There are thirty questions. Mark your answers on the computer answer sheet. Use a #2 pencil only. You may use this question booklet for scratch work. You may keep this booklet.

4. Put your name on the Scantron. Your last name must start in the left column of the last name section. You do not need to put blanks at the end of your name. Put your nine-digit student number in the Banner ID on the Scantron sheet. You **must** include the three zeros at the beginning of this number. If your name and number are not on the sheet your answers will not be graded.



1. If the length of each edge of a cube is increased by 5 inches, then the volume is increased by 335 cubic inches. What was the length of each edge of the original cube?

   A) 1 inch

   B) 2 inches

   C) 3 inches

   D) 4 inches

   E) 5 inches

2. Today at the park, you counted a total of 41 wheels on 16 bicycles and tricycles. How many bicycles did you see?

   A) 6

   B) 7

   C) 8

   D) 9

   E) 10

3. A man spent 1/3 of his money and then lost 2/3 of the remainder. He now has $12. How much money did he have at the start?

   A) $18

   B) $27

   C) $36

   D) $54

   E) $72

4. The lengths of two opposite sides of a square are increased by 20%, and the lengths of the other two sides are decreased by 30%. The area of the square is

   A) decreased by 8.4%

   B) decreased by 10%

   C) decreased by 16%

   D) decreased by 60%

   E) the same

5. A goat is tied to a 10 ft long rope which is attached to the midpoint of a 16 ft long fence. How much area does the goat have for grazing?

   A) 50π ft^2

   B) 54π ft^2

   C) 100π ft^2

   D) 104π ft^2

   E) 108π ft^2

6. Two customers enter a restaurant and see 8 empty stools at a counter arranged in a line. If the customers do not want to sit next to each other, then the number of seating arrangements is

   A) 2

   B) 8

   C) 21

   D) 31

   E) 42

7. Poles of heights *a* and *b* stand a distance *d* apart on level ground. Guy wires are strung taut from the top of each pole to the base of the other. The height above the ground of the point where the wires cross is

A) $\frac{d}{2}$

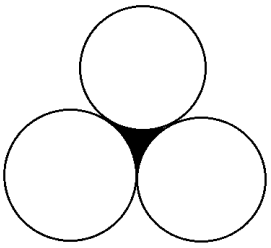
B) $\frac{a+b}{2}$

C) $\frac{ab}{a+b}$

D) $\frac{a-b}{2}$

E) $\sqrt{ab}$

8. Find the area of the shaded region in the figure below if the three circles are mutually tangent and each has radius 2.



A) $4\left(\sqrt{3}-\frac{\pi}{2}\right)$

B) $4\left(\sqrt{3}-\frac{\pi}{3}\right)$

C) $4\left(\sqrt{3}-\frac{\pi}{4}\right)$

D) $4\left(\sqrt{3}-\frac{\pi}{5}\right)$

E) $4\left(\sqrt{3}-\frac{\pi}{6}\right)$

9. Suppose you pull out two socks from a drawer containing 4 blue socks, 2 red socks, and 2 green socks. What is the probability that you have a matching pair of socks?

   A) 1/8

   B) 2/7

   C) 3/8

   D) 1/2

   E) 4/7

10. Let *A*, *B*, and *C* be the midpoints of the sides of an equilateral triangle. Shade in triangle *ABC*. Repeat this process for the remaining un-shaded equilateral triangles. If you repeat this process indefinitely, what fraction of the original triangle is shaded?

    A) 1/5

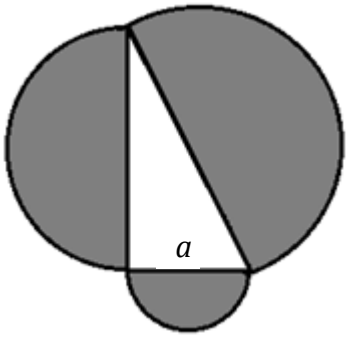
    B) 1/4

    C) 1/3

    D) 1/2

    E) 1/1

11. A right triangle is drawn with base a and height $2a$. On each side of the triangle, a semi-circle is drawn with a diameter equal to that side of the triangle. Find the area of the combined semi-circles.



A) πa^2

B) $\sqrt{5}\,\pi a$

C) $\frac{\sqrt{3}}{2}\pi a$

D) $\frac{5}{4}\pi a^2$

E) $\frac{9}{2}\pi a^2$

12. Consider n switches, each with an up position and a down position, that control a single light. The on/off status of the light can be changed by flipping a single switch. Initially, all of the switches are in the up position and the light is turned on. How many combinations of up/down positions are there so that the light is turned on?

A) $\frac{n!}{2}$

B) $n!$

C) $\frac{n!}{3}$

D) 2^{n-1}

E) $\frac{2^n}{3}$

13. When a four digit whole number is divided by 141 the remainder is 60, and when it is divided by 140 the remainder is 80. What is the remainder when it is divided by 50?

A) 30

B) 40

C) 50

D) 60

E) 70

14. In the triangle ABC below, suppose that AD = DB, BE = FC = ¼ BC, and CG = ¼ AC. If the area of ABC is 32 cm^2, then what is the area of the shaded region?

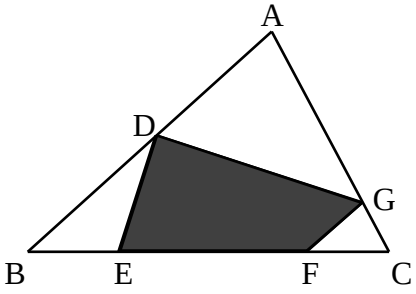
A) 10 cm^2

B) 12 cm^2

C) 14 cm^2

D) 16 cm^2

E) 18 cm^2



15. A cube with side length 5 cm has a rectangular box cut out of each of its six faces. Each cutout box has a square base with side length 1 cm and height 2 cm. Find the surface area of the remaining solid.

A) 150 cm^2

B) 188 cm^2

C) 192 cm^2

D) 196 cm^2

E) 198 cm^2

16. Suppose you want to use 5 different colors to fill in rectangles A, B, C and D in the diagram below. The only restriction is that the rectangles which share a side cannot have the same color. In how many different ways can you to fill in the diagram?

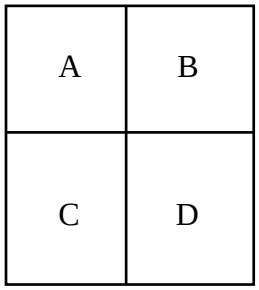
A) 180

B) 240

C) 260

D) 320

E) 625



17. Four people wish to form a touch football team consisting of a quarterback, a pass receiver, a center, and a blocker. How many different teams can be formed if the owner of the ball (who is one of the four people) must be either a receiver or a quarterback?

A) 1

B) 6

C) 12

D) 24

E) 256

18. In triangle *ABC* shown below, suppose that $AD = DB$, $AC \perp BC$, and $DE \perp AB$. If $AB = 20$ and $AC = 12$, then what is the area of *ADEC*?

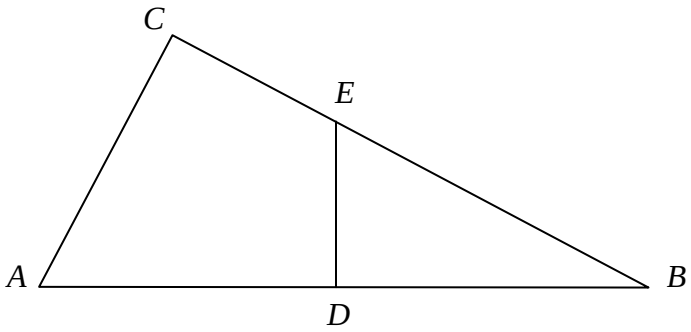
A) 37 ½

B) 48

C) 58 ½

D) 75

E) 96



19. Ten pennies are arranged in two stacks, one with 6 pennies and the other with 4. We remove one coin from each stack and use them to create a new stack. Then we repeat this operation until we reach an arrangement which never changes (the order of the stacks is unimportant). What is that arrangement?

A) 4, 4, 2

B) 5, 5

C) 5, 3, 2

D) 2, 2, 2, 2, 2

E) 4, 3, 2, 1

20. The circumference of the rear wheel of a wagon is 1 foot greater than the circumference of the front wheel. If the front wheel makes 22 more revolutions in a mile (5280 feet) than the rear wheel, what is the circumference of the front wheel?

A) 14 feet

B) 15 feet

C) 16 feet

D) 14π feet

E) 15π feet

21. Note that a square has 2 diagonals and a regular pentagon has 5 diagonals. How many diagonals does a regular 10 sided figure have?

A) 35

B) 40

C) 45

D) 50

E) 70

22. The equation $x + \sqrt{x + \sqrt{x + \sqrt{x + \cdots}}} = 4$ has exactly

A) one solution, $x = 1$

B) one solution, $x = 2$

C) one solution, $x = 3$

D) two solutions, $x = 1$ and $x = 3$

E) two solutions, $x = 2$ and $x = 6$

23. In a survey, 115 high school students were asked whether they liked or disliked algebra, geometry, and trigonometry. 20 students liked all three subjects, while 24 disliked all three subjects. 59 students liked at least trigonometry, 41 disliked geometry but liked at least one of the other two subjects, and 27 liked exactly two of the three subjects. If 15 students liked only geometry and 11 liked only algebra, then how many students liked only trigonometry?

A) 6

B) 9

C) 12

D) 15

E) 18

24. Let $y_1 = f(x) = \frac{x+1}{x-1}$ and, for $n = 2, 3, 4, \ldots$, let $y_n = f(y_{n-1})$. Find y_{2010}.

A) x

B) $x + 1$

C) $x - 1$

D) 1

E) –1

25. Recall, if n is a positive integer, then $n! = n(n-1)\cdots 2 \cdot 1$. Find the greatest common factor of 10! and 10^{10}.

A) 10

B) 800

C) 1,600

D) 6,400

E) 25,600

26. Addison and Juan are playing a coin toss game in which Addison gets one point for the appearance of a head and Juan gets one point for the appearance of a tail. What is the probability that the game will end in a tie after 6 tosses?

A) 1/4

B) 5/16

C) 3/8

D) 7/16

E) 1/2

27. Let d be the largest positive integer that leaves a remainder of 1 when divided into 631, 911, and 1051. Find d.

A) 5

B) 7

C) 10

D) 70

E) 210

28. John, Lucy, and Monica need 15 hours to finish a project together. Suppose John, Lucy, and Monica worked together for 6 hours, and then Lucy quit. It took John and Monica 20 hours to finish the rest of the project. What portion of the original project did Lucy finish before she quit?

A) 1/20

B) 7/50

C) 9/50

D) 11/50

E) 3/10

29. The expression $\sqrt{5+2\sqrt{6}}-\sqrt{5-2\sqrt{6}}$ equals

A) $2\sqrt{2}$

B) $2\sqrt{6}$

C) $4\sqrt{2}$

D) $4\sqrt{6}$

E) $2\sqrt{2\sqrt{6}}$

30. In the parallelogram ABCD shown below, suppose that DE = EC, and that the area of the shaded region is 2 cm^2. What is the area of the parallelogram?

A) 16 cm^2

B) 18 cm^2

C) 20 cm^2

D) 22 cm^2

E) 24 cm^2